



**Are the scattering and the absorption coefficients two faces of the same coin?  
Reverberation time in two cases analyzed.**

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The scattering coefficient is intended to be used in room acoustic calculations and simulation/auralization models. The scattering coefficient,  $d$ , defines the fraction of the scattered sound that is uniformly diffused with smaller energy relative to the specular energy.

This loss of sound energy taking place by dispersion may give the impression that the material upon which the incident sound impinges has a seemingly bigger absorption that it is obtained only by applying the statistical coefficient of absorption. Therefore, in computer programs for room acoustic simulation it appears that the scattering coefficients are generally adjusted to conform to the desired values of reverberation times that were previously well known from other possibly analytic methods, that reflect the necessity of really producing dispersion. We ask: what effect does diffuse reflection have on RT? Will diffuse reflection always affect the RT? In this sense, will acoustic room simulation solve the problem?

In this communication we shall try to clarify a bit more on how much the problem RT calculation depends on “true absorption” or on “apparent absorption”, in where diffusion may included in the sound as an absorbent effect. We will execute RT calculations for two real cases, using PC acoustic simulation of a room and analytic methods, and we will examine the results.

## **1. INTRODUCTION**

The scattering coefficient  $d$ , defines the fraction of the scattered energy that is uniformly diffused. It is defined as follow:

$$d = E_{diff} / E_{total\ reflected} = E_{diff} / E_{spec} + E_{diff} \quad (1)$$

where:  $E_{diff}$ ,  $E_{spec}$  and  $E_{total\ reflected}$  represent the sound energies diffused, specular and total reflected respectively.

Surface diffusion and edge scattering occur as wave phenomena that are usually badly misrepresented by room acoustics software, on the basis of geometrical acoustics hypothesis.

In reality the diffusers are usually applied in situations where some sources and receivers are positioned in the near field. If such is the case, then measurements should be carried out in both the far and near fields. Far field measurements monitor diffusion, while measurements in the near field monitor aberrations, particularly focusing. The far field condition is obtained if the distances  $r$  from the source and receiver fulfil the following requirements:

$$r \gg L_{\max}, \quad r / L_{\max} \gg L_{\max} / \lambda$$

where  $L_{\max}$  is the largest dimension of the diffuser,  $\lambda$  is the wavelength and  $r$  is the distance from either the source or receiver to the measurement position.

## 2. DISCUSSION REGARDING SCATTERING COEFFICIENT AND SOUND ENERGETIC BALANCE

We describe here an approach to evaluate the degree to which a potential diffusing surface uniformly scatters sound. There is the diffusion coefficient  $\alpha$  that defines the fraction of the scattered energy that is diffused. Figure 1 illustrates the normalized incident energy, denoted by a 1, the scattered sound, denoted by  $(1-\alpha)\delta$ , and the specularly reflected energy, designated by  $(1-\alpha)(1-\delta)$ .

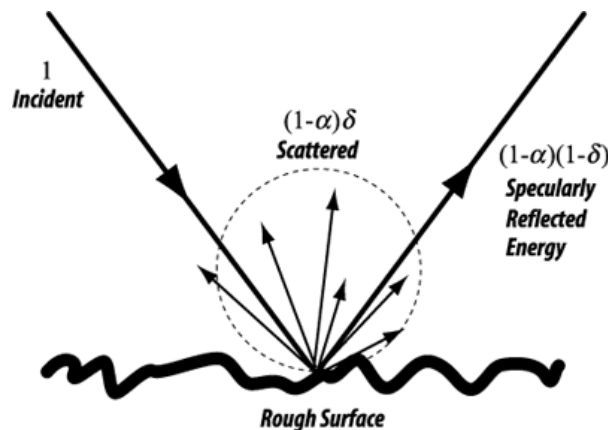


Figure 1. Types of scattering from a rough surface [1]

If  $E_i$ ,  $E_r$ , and  $E_a$  are the sound energy incident, reflected and absorbed, we have, according to the law of conservation of energy:

$$E_i = E_r + E_a$$

or

$$E_i = (1-\alpha) E_i + \alpha E_i,$$

Defining:

$$(1 - \alpha) E_i = (1 - \mathbf{d}) E_r + \mathbf{d} E_r$$

we obtain:

$$(1 - \alpha) = (1 - \mathbf{d}) E_r/E_i + \mathbf{d} E_r/E_i$$

$$(1 - \alpha) = (1 - \mathbf{d})(1 - \alpha) + \mathbf{d}(1 - \alpha) \quad (2)$$

From the above expression we have derived an expression for  $(1 - \alpha)$ :

$$(1 - \alpha) - (1 - \mathbf{d})(1 - \alpha) = \mathbf{d}(1 - \alpha) \quad (3)$$

Mommertz and Vorlander, [2], suggested the following **identity**:

$$1 - \mathbf{a} = (1 - \mathbf{d})(1 - \alpha) \quad (4)$$

where  $\mathbf{a} = 1 - R_{\text{spec}}^2$  and  $R_{\text{spec}}$  denotes the specular reflection coefficient. In this case  $\mathbf{a}$  is called "**pseudo-specular absorption coefficient**" and  $R_{\text{spec}}$  is termed the associated specular reflection coefficient.

The scattering coefficient does not include any information about the directivity of the scattered energy. From equations (2) and (4), the scattering coefficient,  $\delta$ , can be easily determined by:

$$\mathbf{d} = \mathbf{a} - \alpha / 1 - \alpha, \quad (5)$$

In this case it is observed that if  $\mathbf{d} = 1$ , it follows that  $\mathbf{a} - \alpha = 1 - \alpha$ , and therefore  $\mathbf{a} = 1$  must result. What does this mean? Does the effects of the sound scattering increase the absorption of the material?

Substituting (4) in (3), we have:

$$(1 - \alpha) - (1 - \mathbf{a}) = \mathbf{d} (1 - \alpha)$$

or

$$\mathbf{a} - \alpha = \mathbf{d}(1 - \alpha)$$

where if  $\mathbf{d} > 0$  and  $(1 - \alpha) > 0$ , then:  $\mathbf{a} > \alpha$  (except when  $\mathbf{d} = 1, \alpha > 0$ , with  $\mathbf{a} = 1$ ), or when  $\mathbf{d} = 0$  and  $(1 - \alpha) > 0$  where  $\mathbf{a} = \alpha$  is obtained. Therefore, when we have  $\mathbf{a} = 1$  for the case of  $\mathbf{d} = 1$  and if  $\mathbf{d} = 0$ , it follows that  $\mathbf{a} = \alpha$  for all values where  $0 < \mathbf{d} < 1$ , while  $\mathbf{a} > \alpha$  always applies.

The result of this analysis demonstrates that the coefficient  $\mathbf{a}$  is an energy absorption coefficient is possible only for border conditions, and it is an special Sabine type “absorption coefficient exponent”, type Sabine:  $\mathbf{a} = -\ln(1 - \alpha)$ , for all  $\alpha$  lying in the intermediate interval  $\mathbf{a} = 1 > \alpha > 0$ , with the result that  $\mathbf{a} \sim \alpha$  for a low values of  $\alpha$ , under which the law of conservation of the energy can be violated; and, moreover, may incorporate an anomalous dependence on the false coefficient of absorption posing a rather misleading confusion with the diffusion coefficient, in that it does not relate with the absorption coefficient of the material.

As a **first conclusion**, we observe that the identity (4) introduces an acoustical fallacy and a tendency to generate confusion between absorption and sound scattering or diffusion, with an anomalous dependence on  $\mathbf{a}$  and  $\mathbf{d}$  (which, we think, must be independent of the absorption material coefficient because  $\mathbf{d}$  should be dependent on the geometry of the surface).

### 3. SCATTERING COEFFICIENT AND GEOMETRY OF DIFFUSER.

When diffusion is strongly related to the ratio between surface roughness size, or the edge effects are included, it is obvious that the scattering phenomena obeys the geometry of the surfaces upon which the sound impinges, so that we have:

a) wavelength  $\lambda \gg L$ : low diffusion, b) wavelength  $\lambda \sim L$ : high diffusion, and c) wavelength  $\lambda \ll L$ : geometrical mixing, occurring as diffusion.

The diffusion coefficient may vary more strongly with frequency than with its corresponding absorption coefficient. This behaviour may also be true with respect to the scattering coefficient. This frequency dependence indicates that for a given wavelength of the sound in the surface reliefs of the material, irregularities and edge effects, it would be noted that each one of them produces a similar resonance effect of a resonance will take place. The first main distinction, that needs to be made, is the distinction between edge scattering and surface diffusion. In the practice, both phenomena are not always distinguishable from each other, because often diffusion surfaces can be visualized as being many smaller ones presenting a lot of edges, so that it is not clear if the diffusion is produced by the surfaces or by their edges.

A flat polished surface can cause specular reflections and diffusion can occur only at its edges, and also if there is a discontinuity, or free edge. Therefore we feel that both phenomena are regulated by a same diffraction law of diffraction that we propose here:

$$d_i = 10^{-\left[1 - \left(\frac{f}{f_{hi}}\right)^2\right]^{1/2}}, \text{ for } i=1\dots n \quad (6)$$

where  $f_{hi} = c/L_i$  are limiting frequencies of each single relief in the surface under consideration, with  $L_i$  representing the dimension typical of each one them, and  $f = c/\lambda$  is the sound frequency, and  $|\cdot|$  is absolute value of the expression enclosed. If for example, in one surface of area  $S$  there are 4 reliefs with different areas  $S_1, S_2, S_3,$  and  $S_4$  of same material  $\alpha$ , with  $S_1 + S_2 + S_3 + S_4 = S$ , then the overall scattering average will be given by the following expression:

$$\bar{d} = d_1 \left( \frac{S_1}{S} \right) + d_2 \left( \frac{S_2}{S} \right) + d_3 \left( \frac{S_3}{S} \right) + d_4 \left( \frac{S_4}{S} \right) \quad (7)$$

Let the three magnitudes of three relief lengths be represented by  $L_1, L_2, L_3, L_4$  and two in vertical sense, where  $L_1$  and  $L_2$  occupies the same area  $S_1$ ,  $L_3$  and  $L_2$  occupies the area  $S_2$ , and  $L_4$  in both senses occupy the area  $S_3$ . Because of this geometry we have (7) is written as follows:

$$\bar{d} = (d_1 + d_2) \frac{S_1}{S} + (d_2 + d_3) \frac{S_2}{S} + d_4 \frac{S_3}{S} \quad (8)$$

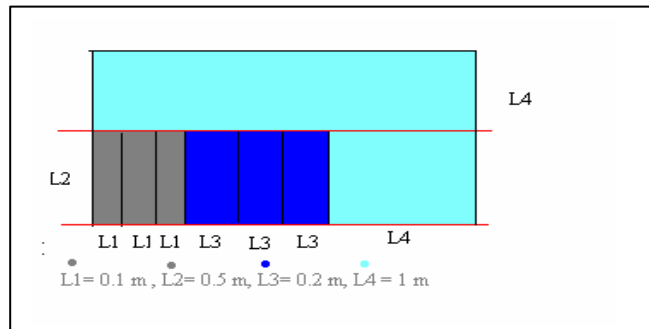
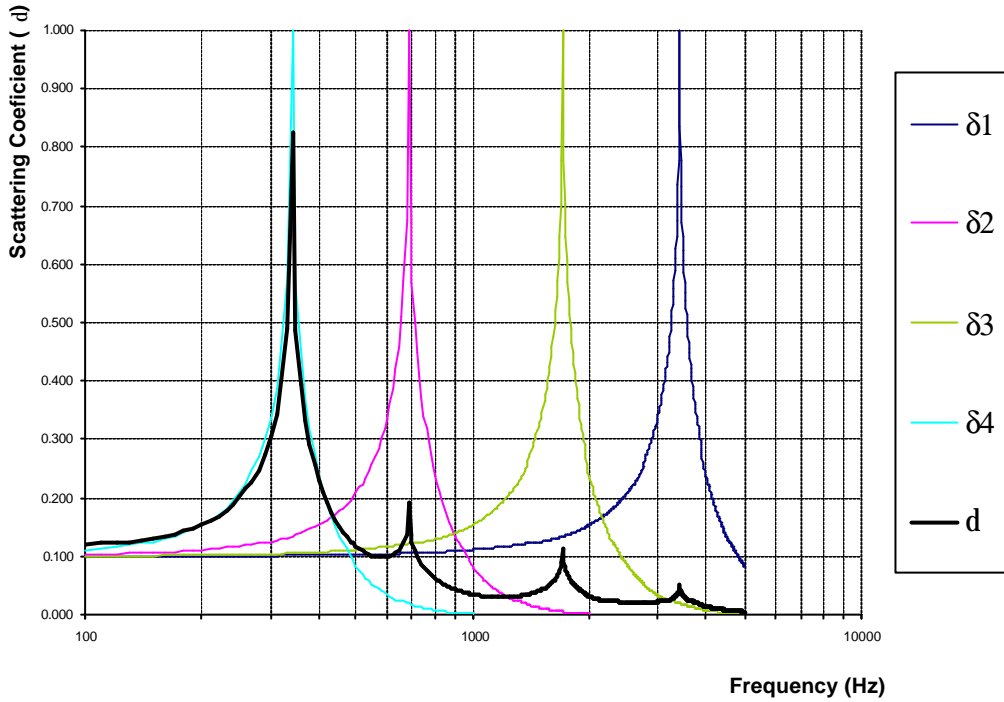


Figure 2: example of relieves sound scattering

Frequency Hz	125	250	500	1000	2000	4000
$d_1$	0.100	0.101	0.103	0.111	0.155	0.240
$d_2$	0.104	0.117	0.210	0.084	0.002	0
$d_3$	0.101	0.103	0.111	0.155	0.240	0.069
$d_4$	0.117	0.210	0.084	0.002	0	0
$d_1 + d_2$	0.204	0.218	0.313	0.195	0.157	0.240
$d_2 + d_3$	0.205	0.220	0.321	0.239	0.242	0
$d_4$	0.117	0.210	0.084	0.002	0	0
$(d_1 + d_2) S_1/S$	0.011	0.012	0.017	0.010	0.008	0.013
$(d_2 + d_3) S_2/S$	0.022	0.023	0.034	0.025	0.025	0
$(d_4) S_3/S$	0.092	0.166	0.066	0.002	0	0
<b><math>d</math></b>	<b>0.13</b>	<b>0.201</b>	<b>0.117</b>	<b>0.037</b>	<b>0.033</b>	<b>0.013</b>

Being:  $L_1=0.1\text{m}$ ,  $L_2=0.5\text{ m}$ ,  $L_3= 0.2\text{ m}$  and  $L_4= 1\text{ m}$ ,  $S = 2.85$ ,  $S_1/S = 0.053$ ,  $S_2/S = 0.105$ ,  $S_3/S = 0.789$ ;  $f_{h1}=3400\text{ Hz}$ ,  $f_{h2}=680\text{Hz}$ ,  $f_{h3}=1700\text{ Hz}$ ,  $f_{h4}=340\text{ Hz}$



**Figure 3:** Scattering coefficient versus frequency for the case of example, where  $\bar{d}$  the average Scattering ( $\bar{d}$ ) calculated using (7) and (8) from individual values  $d_i$ .

From the obtained results we note how many areas of the flat panel are greater than other those of the non-flat area fractions so that the averaged scattering is dominated by the flat panel.

In order to ensure the highest geometrical accuracy where it is needed, the lower the absorption coefficient and the higher the diffusion coefficient, the more surface sources are used.

The strength of each diffused reflection, according to Lambert, is proportional to the cosine of the incidence angle as well as the cosine of the reflection angle, as measured towards the surface normal. However we consider uniform diffusion strength that is dependent on the position of the diffuser in the hall:

$$I_{diff} = \int_s \frac{WQ_q}{4r_1^2} \cdot \cos f_1 \cdot \frac{(1-a)\bar{d}}{2r_2^2} \cdot Q_{f_2} \cdot dS \quad (9)$$

Where

$F_1$  is the angle between the local normal to the surface and the ray arriving from the sound source, being  $r_1$  the distance to source.

$F_2$  is the angle between the local normal to the surface and the ray going to receiver, with  $r_2$  representing the distance to receiver.

$Q_{F_2}$  is the directivity factor in direction  $F_2$ ,

$Q_{F_2}$  assuming that uniform diffusion is equal 1, 2, 3 or 4, according to the position occupied by diffuser inside the hall; (or is varied with  $2\cos F_2$ , in accordance with the Lambert law). (The most common diffuse reflection distribution function is that of Lambert law but many voices claim saying it is not a good function to use).

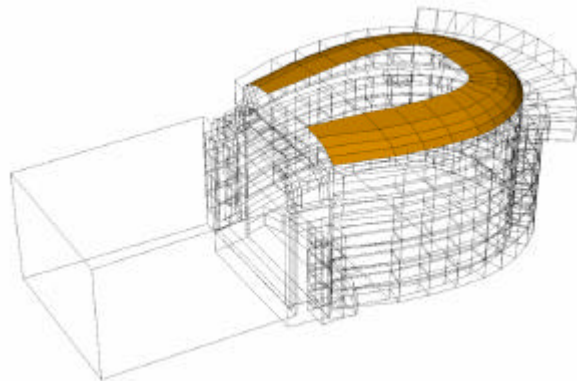
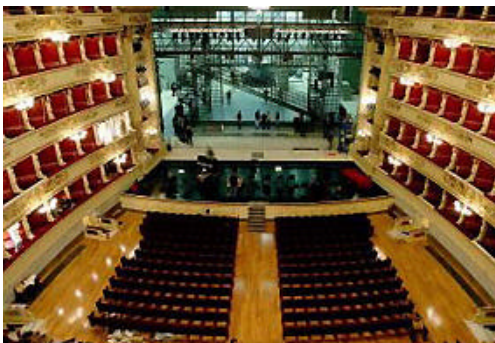
$\bar{d}$  is the average scattering given by (7) and (8)

S is the total area of the reflecting surface

#### 4. REVERBERATION TIME IN TWO REAL CASES ANALYZED.

##### CASE 1. Teatro Alla Scala Milano

Recently we have realised the acoustic improvement of this famous opera house. In this work we used our own software and analytic theories and also other modern and notable room simulation software ODEON for comparison.



CASE 1: The calculations were carried out for the case of the stage complete with setting and side walls and ceiling of stage with backdrop curtains of the Knudsen type absorption type, for an occupied hall. These calculations have been rendered possible by application of the Lambert diffusion hypothesis.

a) Maximum scattering (audience 0.7, remainder walls 0.95)

Frequency (Hz)	125	250	500	1000	2000	4000	$T_{MID}$	$T_{LOW}$	$T_{HIGH}$
RT 20	2.10	1.68	1.35	1.17	1.17	1.17	1.26	1.89	1.17
RT 30	2.08	1.68	1.38	1.20	1.20	1.18	1.29	1.88	1.19

b) Medium scattering (audience 0.7, remainder walls 0)

Frequency(Hz)	125	250	500	1000	2000	4000	T <sub>MID</sub>	T <sub>LOW</sub>	T <sub>HIGH</sub>
RT 20	2.19	2.01	1.89	1.68	1.62	1.44	1.79	2.10	1.53
RT 30	2.52	2.50	2.48	2.24	2.10	2.10	2.36	2.51	2.11

c) Minimum scattering (audience 0, remainder walls 0)

Frequency (Hz)	125	250	500	1000	2000	4000	T <sub>MID</sub>	T <sub>LOW</sub>	T <sub>HIGH</sub>
RT20	2.43	2.28	2.13	1.86	1.92	1.71	2.00	2.36	1.82
RT30	2.80	2.80	2.78	2.48	2.44	2.02	2.63	2.80	2.23

d) Same case with Classical Theories:

Frequency (Hz)	125	250	500	1000	2000	4000	T <sub>MID</sub>	T <sub>LOW</sub>	T <sub>HIGH</sub>
RT Sabine	1.52	1.44	1.28	1.14	1.14	1.13	1.21	1.48	1.14
RT Arau	1.46	1.42	1.31	1.14	1.14	1.14	1.22	1.44	1.14

CONCLUSION 1: The PC-simulated halls ODEON need to include considerable scattering so that the RT result is similar to those of the classical theories (calculated analytically).

If using scattering seems justified in this case, we then analyse the other case, with very flat walls, which represents a situation opposite to that of the first one.

### CASE 2: Vilaseca Auditorium.



Volume: 3443 m3  
 Number audience: 378 seats  
 Condition: Occupied Hall

a) RT with Maximum scattering (audience 0.7, remainder walls 0.95)

Frequency (Hz)	125	250	500	1000	2000	4000	T <sub>MID</sub>	T <sub>LOW</sub>	T <sub>HIGH</sub>
RT 20	1.92	1.74	1.68	1.68	1.74	1.65	1.68	1.83	1.70
RT 30	1.92	1.72	1.68	1.68	1.74	1.64	1.68	1.82	1.69

b) RT with Minimum scattering (audience 0, remainder walls 0)

Frequency (Hz)	125	250	500	1000	2000	4000	T <sub>MID</sub>	T <sub>LOW</sub>	T <sub>HIGH</sub>
RT20	2.22	2.04	1.95	1.80	1.80	1.74	1.88	2.13	1.77
RT30	2.40	2.22	2.20	2.04	2.06	1.90	1.68	2.31	1.98



c) RT measured and RT Classical Theories:

Frequency (Hz)	125	250	500	1000	2000	4000	T <sub>MID</sub>	T <sub>LOW</sub>	T <sub>HIGH</sub>
RT measured	1.83	1.69	1.69	1.65	1.54	1.48	1.67	1.76	1.51
RT Sabine	1.82	1.71	1.65	1.59	1.37	1.38	1.62	1.77	1.37
RT Arau	1.78	1.68	1.71	1.71	1.47	1.42	1.71	1.70	1.45

CONCLUSION 2: In all cases we looked at, in the classical theories, calculated analytic procedure, yield results nearer to measured result. The PC -simulated halls need to include considerably more scattering so that the result for RT is similar to that measured. It may be very strange and contradictory in this case, because all walls are very flat.

## 5. CONCLUSION

1. We have seen that the so-called "**pseudo-specular absorption coefficient**"  $a$ , proposed by Mommertz and Vorlander, is a rather extraneous absorption coefficient that is dependent on and increases with scattering coefficient. This coefficient is highly suspect
2. We have proposed another type of coefficient as the result of analysis of a more geometrically dependent scattering with additional dependence on the frequency and the dimensions of the surface irregularities.
3. In the usual programs of room acoustic simulation we have noted that it appears that the scattering coefficients are being used to adjust the reverberation time to values desired, usually those that were well-known or obtained by another method, possibly analytic, that really needs to consider the effect of diffusion.
4. Therefore we ask: What effect does diffuse reflection have on RT? Will diffuse reflection always affect RT? In this regard, can the acoustic room simulation solve the problem?:

As derived from our results in the above examples, we have seen that RT needs to include scattering in order to result in its value calculated by simulation to agree with the value of RT obtained experimentally..

Are the scattering and the absorption coefficients two faces of a same coin? .....

What do you think?

## 6. BIBLIOGRAPHY

- [1] rpg research: [www.rpginc.com/research/5i4.htm](http://www.rpginc.com/research/5i4.htm)  
 [2] Vörländer M., Mommertz E., Definition and Measurements of Random – incidence Scattering coefficients. Applied Acoustics 60 (2000). S. 187 – 199.