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5aAAb4. The reverberation radius in an enclosure with asymmetrical absorption distribution

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This paper reviews the concept of the reverberation radius from the viewpoint of the classic theories of Sabine and Eyring. These theories are only valid when the sound field is uniformly distributed, or in other words, when the energy density is constant throughout a room. Nevertheless, these theories have also been applied to any spatial sound diffusion situation. For example, they are currently used in rooms with asymmetric absorption distribution, which is generally produced wherever there are asymmetric absorption profiles within the space. This paper proposes a solution to calculate the reverberation radius in rooms with non-uniformly distributed sound absorption (rHND).

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1. INTRODUCTION

The reverberation radius in a room, or critical distance as it is often called in audio engineering, is the distance from a sound source at which the level of direct sound equals the reflected sound level. This distance is an important parameter for the sound perception in rooms, as studies have often proved that the listener must preferably be inside the reverberation radius around the sound source [1]. Available formulas to calculate the reverberation radius referee to sound diffuse spaces. According to these, the value of the reverberation radius (r_{HD}) is the same in any direction within the room. These formulas are based on the classic theories by Wallace Sabine [2] and Carl Eyring [3]. These state that sound pressure variance is zero in a diffuse sound space and the sound energy density is constant. In these cases, the free sound path within the room is equal to lm=4 V/S, where V is the volume of the room and S its total surface.
Conversely, in a no diffuse sound field, sound energy density is not constant; therefore the fluctuations in the sound pressure level depend on the considered direction. A non uniform distribution of absorption in the space often is the main reason for a no diffuse sound field. Moreover, other phenomena, normally wave-type, such as resonance, interference, and focalization may produce privileged sound-wave directions avoiding the sound to diffuse across the volume uniformly. As a consequence of the inhomogeneous distribution of the sound energy, the reverberation radius is not constant among the directions, and the known formulas for calculating the reverberation radius are not anymore valid. The present study proposes a solution to calculate the reverberation radius in rooms with non-uniformly distributed sound absorption (r_{HND}).
This paper is structured in the following way: sections 2 and 3 discuss the theories of diffuse and no diffuse sound field respectively, and show the laws to calculate the reverberation radius in these rooms; section 4 applies the different formulas in a few case studies; section 5 discusses the implications of the proposed formulas for the reverberation radius on the revised theory of sound decay.

2. THE CLASSIC DIFFUSE SOUND FIELD THEORY

Sound level intensity is the sum of the direct sound intensity I_{direct} and the integrated (or diffuse) intensity $I_{diffuse}$, from time t_0 to infinity, as follows:

$$I_{total} = I_{direct} + I_{diffuse} \,. \tag{1}$$

Similarly, sound pressure level may be determined from the intensity level according to:

$$L_p = L_I = 10 \log [I_{total}/I_{ref}] = 10 \log (I_{direct} + I_{diffuse}) / I_{ref} \tag{2}$$

where I_{ref} is the reference intensity, $I_{ref} = 10^{-12}$ Watts/m^2.
We know that the direct intensity can be obtained by the geometrical spreading as:

$$I_{direct} = Qw / 4\pi r^2, \tag{3}$$

where Q is the source directivity, w is the sound power of the source and r is the distance from the source to the receiver.
Assuming the source as omnidirectional (Q=1), we also know that the diffuse intensity is equal to:

$$I_{diffuse} = 4\ w / A, \tag{4a}$$

where A is equivalent absorption, A = [-S ln(1-α)+ 4mV], α is the absorption coefficient, and m is the sound absorption coefficient of the air. At the same time, the diffuse intensity may be expressed as:

$$I_{diffuse} = 25\ w\ (T/V). \tag{4b}$$

From previous equations (3,4), the total sound level in (2) may be expressed as:

$$L_p = L_w + 10 \log (1/4\pi r^2 + (4/A)), \tag{5a}$$

$$L_p = L_w + 10 \log (1/4\pi r^2 + (25(T/V)). \tag{5b}$$

Whenever α is low and the air absorption may be ignored (mV≈0), then A = S·α. The formulas (4a) and (4b) lead to the classic Sabine's formula of the reverberation time:

$$T = 0.16\ V / A \tag{6}$$

Other theories regarded as classic have been proposed through the years [4-6], but they have not offered better evaluations of the reverberation time [7-9]. As a consequence, the formula in (6) is still largely used.

Known the expressions for the direct and diffuse sound pressure, it is possible to obtain a formula for the reverberation radius. This comes from equaling the sound intensity in the direct field in (3) and the reflected sound intensity in a diffuse field in (4a):

$$r_{HD} = ((0.01/\,\pi)\,(V/T))^{1/2} \tag{7a}$$

or equally,

$$r_{HD} = (QA\,/16\pi\,)^{1/2}. \tag{7b}$$

As it is evident from the expression (7a), the reverberation radius increases when the reverberation time decreases, or when the total absorption increases.

Formulas (7a) and (7b) show us the distance between a sound source (F) and receivers (R) where the direct field sound level emitted from the source F is equal to the level in the reverberated sound field. In a perfectly diffuse sound field, where the sound source F is emitting sound in all directions, we can suppose that there are infinite points R placed at an equal distance r_{HD} which defines the surface of a sphere (Fig.1). This distance, commonly known as reverberation radius, indicates the points where direct and reflected sound intensities become equal. This means that at any point within the room at which the sound source F is placed, there will be a spherical surface with radius r_{HD}. However, a perfectly diffuse sound field is difficult to create and in fact, real cases with diffuse sound filed (such as sports halls [10] or large reverberant churches [11]) can rarely be considered perfectly diffuse. Moreover, any space which does not comply with the average absorption coefficient condition of $\alpha < 0.2$ may not be considered perfectly diffuse.

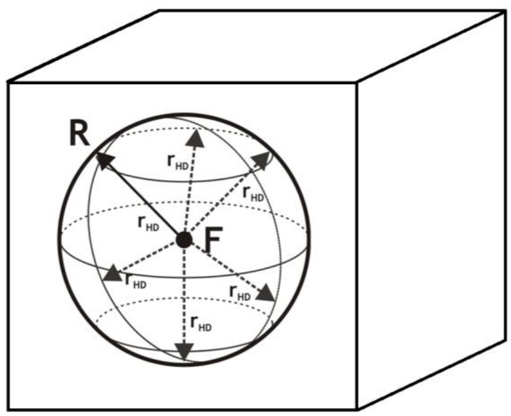

FIGURE 1. Spherical surface of receiver points around a source placed at any point within a room.

3. CLASSIC NON-DIFFUSE SOUND FIELD THEORIES

The first non-diffuse sound field theory was proposed by Dariel Fitzroy [12] in 1959. His proposal became established through an intuitive formula and after repeated experimentations. He based on an earlier idea put forward by Hope Bagenal who stated: "*Reverberation in a rectangular room really consists of three sets of inter-reflections set up between the three pairs of opposite surfaces. It is important that these three reverberation times should be roughly of equal length as one smooth tone*" [13]. However, Bagenal did not indicate which type of mean should be taken into account for the calculation of the reverberation time.

Fitzroy's proposal for the reverberation time T was expressed through the arithmetical mean, weighted by the fraction of area, of the three reverberation times calculated for each direction (x,y,z). In his paper, the direction x connects the floor and ceiling opposing surfaces; direction y connects right and left-hand lateral surfaces; and direction z connects front-back surfaces.

Later, in1988, H. Arau-Puchades, attracted by the intuition of Fitzroy's formula, sought to explore it further [14]. This led him to demonstrate that the reverberation time for a non-diffuse field may be expressed as the geometric average, weighted by the fraction of the area, of three reverberation periods in each direction (x,y,z).

Arau-P worked with a logarithmic-normal distribution of the decay coefficients in the three directions, D_i (i = x,y,z), which were proportional to the absorption coefficients in the three directions (a_x, a_y, a_z). Through this approach, he assumed a distribution of the absorption that does not comply with Gauss's normal law of the mean free path. The resulting reverberation time was hence:

$$T = [T_x]^{Sx/S} \cdot [T_y]^{Sy/S} \cdot [T_z]^{Sz/S}, \quad (8)$$

where each factor T_i (for i = x,y,z) is:

$$T_i = 0.16V/ [-S\ln(1-\alpha_i)+ 4mV].$$

and the absorptions S_i (for i = x,y,z) in each direction are:

$$A_x= [-S \ln(1-\alpha_x)+ 4mV], \text{ where } \alpha_x = (S_{x1}\alpha_{x1}+ S_{x2}\ \alpha_{x2})/S_x$$

$$A_y= [-S \ln(1-\alpha_y)+ 4mV], \text{ where } \alpha_y = (S_{y1}\alpha_{y1}+ S_{y2}\alpha_{y2})/ S_y,$$

$$A_z= [-S \ln(1-\alpha_z)+ 4mV], \text{ where } \alpha_z = (S_{z1}\alpha_{z1}+ S_{z2}\alpha_{z2})/ S_z,$$

where

$$S_x = S_{x1} + S_{x2}; \quad S_y = S_{y1} + S_{y2}; \quad S_z = S_{z1} + S_{z2},$$

$$S=S_x+S_y+S_z,$$

$$\mathbf{A} = A_x^{Sx/S} \cdot A_y^{Sy/S} \cdot A_y^{Sz/S}.$$

Finally, the expression in (8) may be re-written as:

$$T= [0.16\ V / A_x]\ S_x/S \cdot [0.16\ V / A_y]\ S_y/S \cdot [0.16\ V / A_z]\ S_z/S. \quad (9)$$

Moreover, Arau-P also proposed a new expression of the Early Decay Time (EDT), which provided information regarding its average value, but which was not dependent on the particular geometry of each room:

$$EDT = D \cdot d, \quad (10)$$

being D = 60/T, with T calculated according to (8), and d calculated by

$$d= \text{antilog} \cdot \{(S_x/S)(\log a_x)^2 + (S_y/S)(\log a_y)^2 +(S_z/S)(\log a_z)^2 - [(S_x/S)(\log a_x) + (S_y/S)(\log a_y)+(S_z/S)(\log a_z)]^2\}^{1/2} \quad (11)$$

where the absorption exponents are: $a_i = -\ln(1- \alpha_i)$, for i = x, y ,z.
The Arau-P's formulation demonstrated that Fitzroy's theory was either correct or incorrect. In fact, it is valid when the reverberation periods (T_x, T_y and T_z) are equal, or approximately equal, whatever the values of the average absorption coefficients in each direction are.
Consequently, depending on the average absorption coefficient for the space in question, a tendency for coincidence of the Fitzroy's formula with the Sabine's and Eyring's formulas may occur. Reversely, for reverberation periods well differentiated among the directions, the Fitzroy's formula diverges significantly from experimental and theoretical results [7-9].
The formula (9) covers diffuse and no diffuse sound fields, and appears as a general formulation of the theory of reverberation/ In fact, in case of a uniform absorption distribution within the space, it coincides with Sabine or Eyring formulas. Several decades of practices have showed that, in every formula, the accuracy of the previous formulas for calculation of the reverberation time is reduced by the reliability of the absorption coefficients. These inaccuracies also affect software simulations, where in addition limits related to scattering coefficients exist.
From equation (9), it is possible to calculate the reverberation radius in each direction similarly to the expression (7a):

$$r^2_{HNDi} \cdot 16\pi = A_i\ ,\ i =x,y,z, \quad (12)$$

In (12), the reverberation radius in each direction increases as the equivalent absorption in that direction rises. From this, it is possible to calculate the reverberation time as a product of terms of reverberation for the facing surfaces:

$$T=(0.01V/ \pi\ r^2_{HNDx})^{Sx/S} \cdot (0.01V/ \pi\ r^2_{HNDy})^{Sy/S} \cdot (0.01V/ \pi\ r^2_{HNDz})^{Sz/S}, \quad (13a)$$

$$T= (0.01V/ \pi\ r^2_{HND}), \text{ where } r_{HND} = [r_{HNDx}^{Sx/S} \cdot r_{HNDy}^{Sy/S} \cdot r_{HNDz}^{SxzS}], \quad (13b)$$

with the reverberation radii for a non-diffuse sound field r_{HND} are obtained as:

$$r_{HND} = ((0.01/ \pi)\ (V/T))^{1/2}, \quad (14a)$$

$$r_{HND}= (\mathbf{A}/16\pi)^{1/2}, \text{ where: } \mathbf{A} = A_x^{Sx/S} \cdot A_y^{Sy/S} \cdot A_y^{Sz/S}. \quad (14b)$$

Finally, if A_x, A_y, and A_z tend to be equal, which corresponds to have the absorption periods equal in the three directions, then $r_{HND} \approx r_{HD}$. As it is evident, this case reduces the reverberation radius to the expression (7) in the case of diffuse sound fields.

4. ANALYSIS OF THE REVERBERATION RADIUS IN NO DIFFUSE SOUND FIELDS

In this section a comparison among values of the reverberation radii according to different formulas is performed. Figure 2 shows the configuration of the rooms used in the experimental and theoretical inter-comparison among theories for no diffuse spaces carried out by Mehta and Mulholland [7]. This study was the first comparative test about asymmetric distribution absorption in a room. The room had the following dimensions: length 4.5 x width 2.7 x height 2.4 m; its volume was V= 29.16 m^3.

Five of the Mehta-Mulholland's cases have been investigated. Moreover, the simplest possible space, that is the room with no absorptive panels, was considered. This last case was used for a verification of the accuracy of the models and it is here referred as case 0. In the other five configurations, an absorptive material was used to add non-uniform absorption to the room. The investigated rooms are represented in Figure 2:

- Case 1: absorption on the long walls;
- Case 2: absorption on one long wall;
- Case 3: absorption on the floor and the two short walls;
- Case 4: absorption on the floor and on one short wall;
- Case 5: absorption on three mutually perpendicular surfaces.

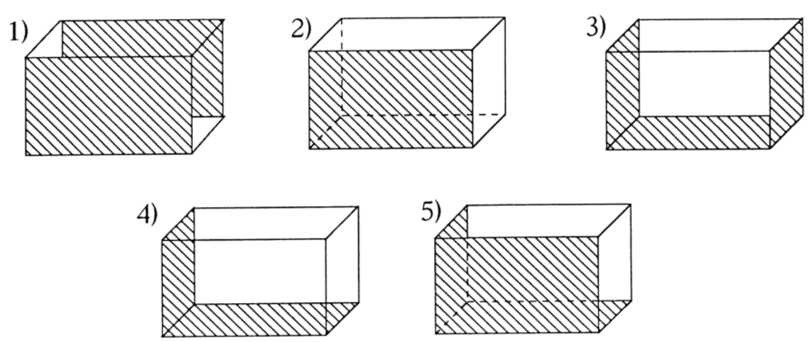

FIGURE 2. Five configurations of room with high absorption marked on walls (dark walls α = 0.86, white walls α = 0.036).

Details of the simulation setup in the different cases, especially for the determination of the reverberation time, can be found in [7]. The results of the reverberation time with the different formulas (using the formulas (2), (3), (13a)) and the reverberation radii (using the equations (7a), (14a)) for the six cases, are represented in Figure 3.

Results of the reverberation radii show that the error committed in the estimation with the formula of Sabine or the formula of Eyring is considerable, being on average 18.5 and 28.9 % of the measured value respectively. However, the estimation with new formula for r_{HND} is much more accurate, and it results in an average error of 2.47% only.

In particular, it is important to underline that all the reverberation radii calculated with the Sabine's or Eyring's formulas exceed the measured values as a consequence of the same problem generally encountered in the "revised theory" [17]. This showed that the level of the reflected sound decreases with the distance from the sound source.

A description of the results in the different cases follows:

- in case 0: it emerges that the different formulas for the reverberation time and the reverberation radius predict similar results; hence all these formulas may be considered to be accurate in the case with no absorptive panel;
- in case 1, it emerges that the Arau-P's formula is significantly more accurate than any of the classical formulas. This demonstrates the limitation of the classical methods, in rooms with a non-uniform distribution of room surface absorption;
- in case 2, it emerges that with absorption on just one long wall, the reverberation time is 0.19 s longer than in case 1, and the reverberation radius decreased from 0.42 m to 0.36 m. These differences are predicted by the three classical formulas, but only the Arau-P's formula and the new r_{HND} are accurate;
- in case 3, the reverberation time reduces to 0.29 s, which is accurately predicted by Arau-P and under-predicted by the other formulas; however, the differences between the methods is larger for the values of the reverberation radius which is underestimated by 33% both with the Sabine's and Eyring's formula;

- in case 4, all the classical models under-predict the reverberation time and overestimate the reverberation radius, whereas the Arau-P's formula results similar to the simulated value;
- in case 5, the Arau-P's and Eyring's formula produce less accurate prediction of the reverberation time than the Sabine formula; this reflects on the estimation of the reverberation radius (a 2% error in the estimation with the classical formula and a 10% with the other ones occur).

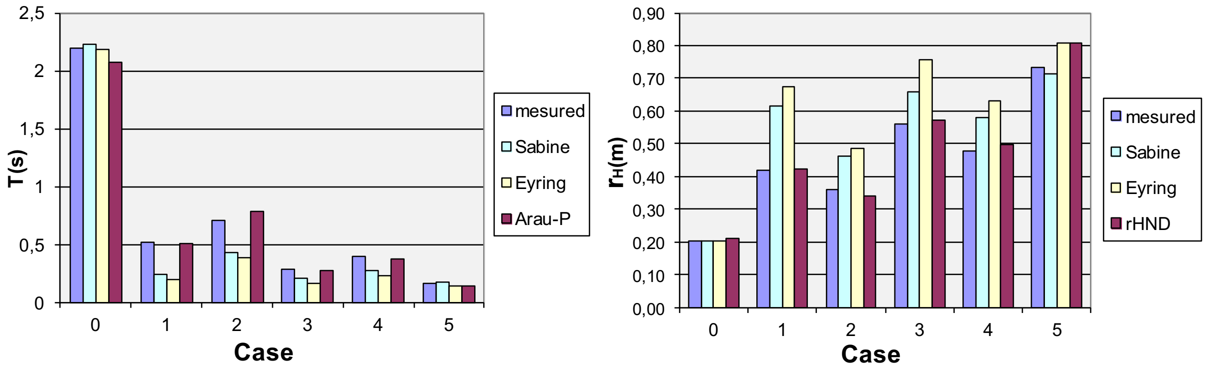

FIGURE 3. Values of the reverberation time (left) (values taken from [7]) and the reverberation radius (right) derived for the cases used in [7]. The case (0) is a room without high absorption.

5 INFLUENCE OF THE REVERBERATION RADIUS ON THE REVISED THEORY

According to the classical theory, sound strength should not vary considerably at sufficient distance from the source where the energy of the reflected sound, which is assumed to be equally distributed throughout the space, is dominant. However, as was shown by extensive objective acoustic measurements by Barron and Lee [17], total reflected sound energy does significantly fall off with increasing source-receiver-distance.
Barron and Lee found that the sound level decay was linear soon after the direct sound in the majority of the halls and the reflected sound level decreased with increasing source-receiver distance. This is due to the fact that receivers closer to the source not only receive a higher level of direct sound but also higher levels of early reflections because they have traveled shorter distances. So, they proposed a model based on the following four assumptions: (i) the direct sound is followed by linear level decay at a rate corresponding to the reverberation time; (ii) the instantaneous level of the late decaying sound is uniform throughout the space; (iii) the time $t = 0$ corresponds to the time the signal is emitted from the source, therefore the direct sound reaches a point at a distance r from the source after a time $t_D = r/c$. In this way the integrated energy decreases when the source-receiver distance increases, while the early/late reflected energy ratio remains constant; (iv) the integrated value for the reflected sound level is assumed to be, at $r = 0$, equal to the value predicted by the classical theory [11]. According to the Barron and Lee's revised theory of sound decay, the sound energy may be calculated as

$$G = 10 \log_{10}(100/r^2 + 31200 \cdot RT/V \cdot e^{(-0.04 \cdot r/T)}). \tag{15}$$

The exponential term $e^{(-0.04 \cdot r/T)}$ marks the difference between traditional and revised theory. It accounts for the fact that the linearly decaying reflected sound, which is assumed to have a uniform instantaneous level at late time, cannot start before the arrival time of the direct sound $t_0 = r/c$, thus yielding a refined integration limit for the calculation of the total reflected sound level.

Although applying the revised theory to concert halls markedly improves the prediction quality compared to traditional theory, the theory still has obvious limitations that have been widely discussed [11, 18]. In particular, there has been some discussion about the appropriate starting time t_0 for the integration. Vorländer suggested that the integration should not start at the arrival time of the direct sound but at the arrival time of the first order reflections, and he showed that the lower limit of integration is identical to the mean free path [18].
Another reasonable assumption for the starting time of the integration has been the direct sound delay plus the delay of the first order reflections (ITDG). Barron [18] acknowledges that considering the ITDG might be beneficial and possibly offer more accurate predictions, but he points out that this would require additional input parameters such as consideration of the shape and geometry of the hall and the exact source and receiver position. In addition he makes

clear that irrespective of the choice of t_0, precise agreement from a theory like this can generally not be expected since using continuous integration of the energy fractions of the reflected sound can obviously not account for the discrete character of the early reflections. In fact, several other methods with no continuous integration have been developed in recent years (the reader can see [11] for a review).

Interpreting what happens to the sound field after the emission of a sound, it is easy to image that initially only the direct sound exits. Then, the reflection of the sound on the surfaces of the room generates a reflected sound field. However, the direct sound level is higher and prevents perceiving the reverberated sound field. This means that, during a lapse of time that can be approximated to the time required by the sound to travel a reverberation radius, our hearing cannot hear the reverberated field although the first sound reflections have already been formed on the walls of the room. In fact, in our ear the diffused field is only perceived when we are able to hear it. Consequently, in a highly reverberant room (high T corresponds to low r_H), the reverberated field is perceived almost instantly. Reversely, if the space is very dry (low T corresponds to high r_H), it is much harder to hear the reverberated sound field.

Another way to interpret this is by considering that at distances less than the reverberation radius, the direct sound predominates, and at distances greater than the reverberation radius, the diffuse sound field predominates. Consequently, the reverberation radius may be considered as an important instrument for assessing the interval of integration of the sound energy [15]. This leads to rewrite the integral of the diffuse intensity as:

$$\overset{\infty}{\underset{t}{I}} = (w / V)\int_{t_d}^{\infty} e^{-(c\,(t - t_H)\,/4\,V)A}\,dt \tag{16}$$

where the lower limit is $t_d = t - t_H$, for $t_d \geq 0$, with t_H the time corresponding to the sound to travel a reverberation radius. Finally, the equivalent formulas for the diffuse intensity, based on the revised theory, are:

$$I_{diffuse} = (4\,w / A) \cdot e^{-((r - r_H)\,/4V)A}, \tag{17a}$$

$$I_{diffuse} = 25\,w\,(T/V) \cdot e^{-0.04(r - r_H)\,/T}, \tag{17b}$$

$$I_{diffuse} = 312\,(w/4\pi)(T/V) \cdot e^{-0.04(r - r_H)\,/T}. \tag{17c}$$

6 CONCLUSIONS

The concept of reverberation radius has scarcely been used in room acoustics, and unfortunately, it has solely been assessed according to classic theory of diffuse sound field. In fact, available formulas are only valid if the distribution of the absorption of the room is uniform. This paper has proposed a new formula to estimate the reverberation radius in rooms with no-uniform absorption. Finally, the paper has shown that the reverberation radius may be used to correct the diffuse intensity in the revised theory.

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