inter.noise 2012
august 19-22
new york city, usa
Quieting the World's Cities

Different assumptions - different reverberation formulae

Uwe M. Stephenson [a]
HafenCity University Hamburg,
Hebebrandstrasse 1,
22297 Hamburg, Germany

The conditions and assumptions behind different reverberation formulae are often not explained very accurately. So, the reasons for their differences are often hardly understood. In this rather didactical paper a more rigorous definition of the crucial term 'diffuse sound field' is proposed and the relationships to the necessary surface conditions, especially scattering, are discussed. The reasons for the difference between some reverberation formulae, first of all, Sabine's and Eyring's, are analyzed. Also Kuttruff's correction formulae for varying free path lengths and uneven absorption distributions as well as approaches to partially diffuse sound fields with their assumptions are discussed.

1 INTRODUCTION

The term "diffuse sound field" (DSF) is often not explained very accurately - the motivation for this paper. The paper is organized as follows: in section 2 , a more rigorous definition is proposed and the relationships to the necessary surface conditions as absorption and scattering are discussed After some general definitions, and a clarification on the computation of mean free path length (in 3), in section 4, the Eyring and Sabine reverberation formulae will be re-derived. In section 5, the reason for the difference between both formulae will be analyzed by some thought experiments. In section 6, different transition models are discussed, in section 7 further reverberation time formulae and approaches to only partially diffuse sound fields.

First, the general condition of geometric/statistic room acoustics is presupposed that typical room dimensions are large compared with wavelengths such that the analysis may be performed with an energetic sound particle model (and for one frequency band). So, ‘Intensity’ I is here interpreted as an integral over the whole solid angle: $I = \int j d\Omega$, a scalar rather than a vector:

$$I = c\,U \text{ (c= sound velocity, U= energy density)} \quad (1).$$

[a] email: post@umstephenson.de

2 CONDITIONS FOR THE DIFFUSE SOUND FIELD

First, it should be distinguished between theoretical definitions and practical conditions, further between the claim the sound field should be diffuse 'from the start' (strict version) or 'towards the end of reverberation" (tolerant version, conditions in brackets) (see Table 1).

Usually one starts with A (' each direction with same intensity /‘directional diffusivity’). From A follows B (in a room without absorption, the particles don't lose energies, see the lines connecting the clusters in Fig. 1) [1], but not vice versa (consider e.g. the case of a long room evenly filled with rays just in a longitudinal direction, Fig. 2). It should be emphasized by the way: The room does not need to be convex (the argumentation of Fig.1. could be extended by more array-clusters) - ***if*** a diffuse sound field is really given; then also the derived consequences as e.g. the formula for mean free path lengths, are true. However, in non-convex rooms, - e.g. with weakly coupled sub-spaces - the sound field ***is*** hardly diffuse. From B (a volume condition) follows as a surface condition B2 (Fig. 1b).

The surface conditions C+D are necessary but not sufficient for A+B: All surfaces may be totally diffusely reflecting, but the irradiation strengths may be non-constant due to geometry (typically if the absorption distribution over the surface is quite uneven). If just one piece of surface is absorbing, then the sound field closely in front will be not isotropic (Fig. F3a). The same happens, strictly speaking, with only one specularly reflecting peace of surface as producing a mirror image source (Fig. F3b, actually already the existence of the source itself is forbidden.) For the tolerant version of the definition of the DSF ('convergence only in late reverberation') it is sufficient that only an average absorption degree needs to be 'low' (typically it is proposed that for the validity of the Sabine formula a mean absorption degree $\alpha_m < 0.3$ is sufficient) and at least a small piece of surface is a bit unregularly i.e. scattering and hence 'mixing' [2].

Only if (fictively) the surfaces were also interchanging positions (evenly distributed), i.e. totally mixing (D2), then from C+D+D2 follows A+B +B2 i.e. aconstant irradiation strength

$$B = I/4 \tag{2}$$

And only if the room is 'totally mixing' i.e. interchanging energy at every place and time into every direction, then there is no chance that different (exponential) energy decays arise and just one single exponential energy decay with on reverberation time is left (Fig. 4).

A constant B is the condition that the notion 'equivalent absorption area' (used to derive the Sabine formula) i.e. a surface-weighting, makes sense. The factor 1/4 is due to the directional averaging the projection factor $cos(\vartheta)$ over the full solid angle 4π:

$$M = \frac{1}{4\pi}\int_{2\pi} cos(\vartheta)d\Omega = \frac{1}{4\pi}\int_0^{\pi/2}\int_0^{2\pi} cos(\vartheta)\ d\varphi sin(\vartheta)d\vartheta = \frac{1}{2}(sin^2(\vartheta)/2)\Big|_0^{\pi/2} = \frac{1}{4} \tag{3}$$

2.1 The Lambert reflection

With the Lambert reflection, the reflection angle (ϑ) probability density ***p’*** per solid angle is proportional cos(ϑ) , independent from the incidence angle (Fig. 5a):

$$p' =: \frac{dp}{d\Omega} = \frac{cos(\vartheta)}{\pi} \tag{4}$$

(π is the normalization factor for the half-sphere). It can be considered as the ideal scattering characteristics of 'rough surfaces' following from the cos- projection law and the reciprocity principle. In room acoustical computer simulation, the mix of diffuse and specular reflections in reality is often simulated by a 'diffusivity 'or 'scattering' coefficients usually simulated by drawing random numbers [3,4], see Fig. 5b. The scattering coefficient $\boldsymbol{\sigma}$ is defined as the proportion: '(non-geometrically energy)/(total reflected energy)'.

3. AVERAGE QUANTITIES IN A DIFFUSE SOUND FIELD

The core *physical* quantity is the equivalent absorption area

$$\mathrm{A} = \sum \alpha_\mathrm{i}\, \mathrm{S}_\mathrm{i} \tag{5}$$

or the 'mean (surfaces averaged) absorption degree'

$$\alpha_\mathrm{m} \equiv \alpha = \frac{\sum \alpha_\mathrm{i} \mathrm{S}_\mathrm{i}}{\mathrm{S}} = \mathrm{A}/S \tag{6}$$

(S_i= single of N surfaces, *S*= total surface, α_i = absorption degrees).

The other, the *geometric* average quantity, is the mean free path length with its famous formula

$$\Lambda = 4V/S \tag{7}$$

(*V*=volume), which is true even for non-convex rooms, if a diffuse sound field really were given – which is, however, hardly the case then. The same is valid for the other relationships.

3.1 Derivations of the formula for the mean free path length (mfp)

The correct mfp-formulae can be derived strictly obeying conditions A…D2. These shall be explicitly named in the following:

Method a) is utilizing
A) isotropy in *Ω* and B) homogeneity in V and averaging over the inverse mfp, i.e. reflection frequencies ('time average'):

$$\Lambda^{-1} = \overline{l^{-1}}^{V,\Omega} \tag{8}$$

This way, also shown by Kosten [5] is gone with the derivation of the Sabine formula. In a DSF, 'sound particles' lose their identity: 'time = ensemble average' [1]. Therefore the averaging can be performed also over a group of 'parallel' rays ('channels') into an ***absolute*** direction δ representative for all (Fig.6).

$$\Lambda(\delta) = \frac{1}{n}\sum_{i=1}^{n} l_i = \frac{V/q}{Q/q} = \frac{V}{Q(\delta)} \tag{9}$$

The inverse of Eqn. 8 inserted in Eqn. 9 yields

$$\frac{1}{\Lambda} = \overline{\left(\frac{1}{\Lambda(\delta)}\right)}^{\Omega} = \overline{\left(\frac{Q(\delta)}{\mathrm{V}}\right)}^{\Omega} = \frac{S}{4V} \tag{10}$$

as the directional average over all projected surface elements dS (also over their backsides) is dS/4. The average cross section of any volume is always Q=S/4. (See the derivation of the factor 1/4 in Eqn. 3.) The directional average over all ‘absolute’ orientations δ must be equivalent to an average over the local incident angle ϑ relative to the surface normal at any part of the room surface with any orientation. Any 'channel' (in Fig.6) intersects the surface twice, but once from the back side, where the averaging is omitted such that the averaging factor is *M=1/4* (as derived in Eqn. 3). Even if the room is not convex, such that the ‘channels’ may be interrupted into several ones, this argumentation holds.

Method b) utilizes B2) i.e. constant irradiation of *S* and D) (everywhere Lambert law) and direct averaging over the mfp ('ensemble average': one considers the 'fates' of different 'sound particles' simultaneously on the way.)

$$\boldsymbol{\Lambda} = \overline{l\,p'}^{\Omega,S} \tag{11}$$

Different from method ***a***, method ***b*** is related to the surface related conditions for the DSF:

$$\boldsymbol{\Lambda} = \frac{1}{S}\int_S \int_{2\pi} l(\vartheta)\,\frac{cos(\vartheta)}{\pi}\; d\Omega dS \tag{12}$$

Now ϑ is the local angle relative to the normal. As in a diffuse sound field the averaging procedures over the surface and the angle are independent from each other (the Lambert law is valid everywhere). The surface integral in the following is 2V, independent from orientation, and can be separated. Thus

$$\boldsymbol{\Lambda} = \frac{1}{\pi S}\int_{2\pi} d\Omega \int_S \; l(\vartheta)cos(\vartheta)dS \; = \frac{2\pi\, 2V}{\pi S} = \frac{4V}{S} \tag{13}$$

4 RE-DERIVATION OF REVERBERATION FORMULAE

Both reverberation formulae assume a diffuse sound field (especially condition B2 i.e. a constant irradiation of the surface leading to a mean absorption coefficient, E in Table 1.)

4.1. The Eyring formula

Typical is here to consider a ´representative sound particle` (sp) (Table 1, condition F) which, after always a free path length Λ, 'sees' a surface with the absorption degree α_m. The consequence is a stepwise exponential energy decay:

$$E(N) = E_0(1-\alpha_m)^N \tag{14}$$

where E_0 is the start energy and *N* is the reflection number (Fig. 7).
By introducing a mean absorption exponent

$$\alpha_m' = -\ln(1-\alpha_m) \tag{15}$$

Eqn. 14 reads $E(N) = E_0 e^{-N\alpha_m'}$. For N reflections with a mfp *Λ*, the time $t = N\,\Lambda/c$ is needed. Tacitly it is assumed that N is a real number as after a switched off steady sound source the decays overlap and 'smooth' the resulting function $E(t) = E_0 e^{-\alpha_m' ct/\Lambda}$. Using the standard formulation of an exponential decay $E(t) = E_0 e^{-\frac{t}{\tau}}$ the time constant of the sound energy decay is

$$\tau_{ey} = \frac{\Lambda}{c\,\alpha_m'} \tag{16}$$

In such a typical RT formula, the time constant is always the proportion of the mean free path length and an average absorption exponent. The RT for a 60dB decay is then generally

$$T = 6\ln(10)\,\tau \tag{17}$$

Using the normalized value of the sound velocity c=340m/s at 14°C and also the value for the mfp Λ (Eqn. 7) yields the Eyring reverberation time

$$T_{ey} = \frac{6\ln(10)}{c}\frac{4V}{S\,\alpha_m'} \approx 0.163\;\frac{V}{S\,\alpha_m'} \tag{18}$$

Additionally (to the DSF) condition D2 is assumed: (only with a total mixing the sp lose their identity and one representative sp may be assumed, condition F) and: 'the mfp are constant' (condition H) –which is of course wrong: they are varying.

4.2. The Sabine formula

The Sabine formula is not just an approximation of the Eyring formula, it has its own, amazingly different derivation. Neither the model of a sp nor the concept of a mfp is used. Instead, the decay of the total sound energy E(t) (one value everywhere!) is considered aiming at a differential equation. Especially the homogeneity of the energy distribution is assumed, however, tacitly even more, namely that there is simply only one value of *E* at the time *t* (condition G). (This will turn out to be the crucial misunderstanding of the Sabine approach.) With the energy density $U = E/V$, $I = c\,U$, the irradiation strength is $B = I/4 = cU/4 = cE/(4V)$ (Eqn.2). Then the incident energy per time is

$$\frac{dE_i}{dt} = cE\frac{S}{(4V)} \quad (19),$$

and finally the absorbed energy :

$$\frac{dE}{dt} = -E(t)\,c\frac{\alpha_m S}{4V} \quad (20).$$

The solution is an exponential decay with the time constant

$$\tau_{sab} = \frac{\Lambda}{c\,\alpha_m} \quad (21)$$

Inserting again $\Lambda = 4V/S$ yields the famous Sabine RT

$$T_{sab} = 6 \cdot \ln(10)\,\tau_{sab} \approx 0.163\,\frac{V}{A} \quad (22).$$

As the energy is proportional the number of sound particles, analogously to Eqn. 19 the sp impact rate is

$$dN/dt = N/t = cN\,S/(4V) \quad (23)$$

(which would be constant without absorption – an allowed assumption for deriving just a geometric quantity as the mfp). After the time for travelling just a mfp, per definition all sp once have hit the room surface. Hence, inserting $t = \Lambda/c$ into Eqn. 23 yields by the way a prove for the formula for the mean free path length $\Lambda = 4V/S$.

5 WHY IS THE SABINE DIFFERENT FROM THE EYRING FORMULA?

For small α_m,

$$\alpha'_m = -\ln(1-\alpha_m) \approx \alpha_m(1+\frac{\alpha_m}{2}) \quad (24),$$

so comparing both formulae (Eqns. 16 and 21) shows that the difference is just in the order of

$$\frac{T_{ey}}{T_{sab}} \approx (1-\alpha_m/2) \quad (25).$$

The reasons for the difference are different tacit additional assumptions: Eyring assumes a constant mean free path length and a stepwise energy decay, i.e. all sound particles lose a part of their energy at the same time [7].

Sabine assumes only one energy value. This is also absurd as this were only possible if the information about absorption at one part of the surface were spread infinitely fast to everywhere within the room, such that local different energies are 'equalized' as if ‘the sound particles know from each other’. This leads to an effectively smaller energy loss and hence a longer reverberation time than with just one 'representative' sp as with the Eyring theory (Fig.8).

6 TRANSITIONS BETWEEN THE TWO FORMULAE

6.1. From Eyring to Sabine

Starting with the Eyring model, obviously one has to consider the time interval between two reflections $\Delta t = \Lambda/c$ more in detail. The first thinking model is to subdivide it. With an equally distributed time shift of many sound particles, the energy loss may be linearly interpolated, so, after a time $\Delta t/n$, the overall energy loss factor is $(1-\alpha/n)$; allowing an ‘information and energy inter change’ the energy loss after ‘1/n reflection’ would be 'equalized'.

So, after a whole reflection – n such steps - the energy would be multiplied by $(1-\alpha/n)^n$. For $n \to \infty$, the loss factor between two reflections would become

$$f_{sab} = \lim_{n\to\infty}\left(1-\frac{\alpha}{n}\right)^n = e^{-\alpha} \approx 1-\alpha+\frac{1}{2}\alpha^2 \dots \approx 1-\alpha\cdot(1-\frac{\alpha}{2}) \quad (26).$$

$e^{-\alpha}$ is the Sabine energy loss factor for 1 reflection (insert $\tau = \frac{\Lambda}{c\,\alpha}$ and $t = \frac{\Lambda}{c}$ into $e^{-t/\tau}$). The corresponding Eyring value is $\quad f_{Ey} = 1-\alpha = e^{-\alpha'} \quad (27)$

Comparing Eqn. 26 with 27, the 'effective' absorption degrees differ in the first approximation by the factor $(1-\alpha/2\,)$, hence (regarding Eqns. 16 and 21) also the proportion

$$T_{ey}/\,T_{sab} \approx (1-\alpha/2\,) \quad (28).$$

So, the difference between the Eyring and the Sabine formula can be explained by the transition from the stepwise to a continuous absorption.

6.2. From Sabine to Eyring

An idea to describe the opposite transition is to assume that for the energy loss at the surface the energy (considered with the Sabine model) in the middle of the room is relevant. Thus the former diff. Eqn.20 with $\Lambda = 4V/S$ has to be altered to

$$\frac{dE}{dt} = -\frac{c\propto}{\Lambda}E(t-\Delta t/2) \quad (29)$$

where $\Delta t = \frac{\Lambda}{c} = \alpha\tau_{sab}$ is the half of the time interval between two reflections. Assuming, as the first approximation an exponential decay according the Sabine RT,

$$E\left(t-\frac{\Delta t}{2}\right) = E(t)e^{\Delta t/(2\tau sab)} = E(t)\,e^{\alpha/2} \approx E(t)\,(1+\alpha/2) \quad (30.$$

Inserted into the differential. Eqn. 29 yields a differential. Eqn. with a modified absorption factor

$$\frac{dE}{dt} \approx -\frac{c\,\alpha\left(1+\frac{\alpha}{2}\right)}{\Lambda}E(t) = -E(t)/\tau_{sabshift} \quad (31).$$

The new time constant is

$$\tau_{sabshift} = \frac{\Lambda}{c\,\alpha(1+\alpha/2)} \approx \tau_{sab}(1-\alpha/2) \approx \tau_{Ey} \quad (32)$$

Thus, again the Eyring reverberation time is reached.

6.3. Kuttruff's 'repair' of the Eyring formula allowing a spreading of the free path lengths

The new parameter is here the relative variance γ^2 of the free path lengths defined by

$$\gamma^2 = \frac{\overline{l^2}-\bar{l}^2}{\bar{l}^2} \quad (33)$$

where $\bar{l}$ is the mfp (called Λ before), $\bar{l}^2$ its square and $\overline{l^2}$ the average over the squares. (A variance of the absorption degrees is not considered, α is still the mean absorption degree.) Kuttruff's approach [1] is to consider the reverberation as a sum of an infinite number of decays with different RTs, thus, a different number of reflections N within the same time, weighted with different probabilities P_N:

$$E(t) = E_0\sum_{N=0}^{\infty}P_N e^{-\alpha' N} \text{ (normalized by } \sum_{N=0}^{\infty}P_N = 1) \quad (34).$$

By expanding the exponential functions into a Taylor series around the mean value $\alpha'\,\bar{N}$ up to the quadratic order ($\bar{N} = ct/\,\Lambda$ is the average number of reflections) and introducing the variance ${\sigma_N}^2$ of the N reflections, the sum of these many decays can be expressed as one unified decay $E(t) = E_0 e^{-\frac{t''}{\tau}}$ with a new time constant $\tau'' = \frac{\Lambda}{c\,\alpha''}$ with a correction term proportional to the variance ${\sigma_N}^2$. The effective absorption coefficient is then

$$\alpha'' = \alpha'(1-\gamma^2\alpha'/2) \quad (35)$$

This is smaller than the Eyring exponent α'. So, with varying free path lengths the RT is longer than without. Usually, depending of the room shape, for typical proportions from 1:1:1 to 1:10:10, the variance γ^2 is in the order of 0.4….0.6 (as found by numerical experiments). Kuttruff's derivation and Eqn. 35 is only valid for small absorption and /or the very first reflections. The late reverberation with its different non-mixing decays, remains always governed by the weakest single decay, i.e. by the longest partial RT. However, one can see: The variation of reflection moments furthers the 'mixing effect' as it is, different from the Eyring theory, tacitly assumed within the derivation of the Sabine formula. For 'totally' varying free path lengths $\gamma^2 = 1$ and with the expansion $\alpha' = -\ln(1-\alpha) \approx \alpha + \alpha^2/2$ inserted into Eqn. 35 it turns out that

$$\alpha'' = \alpha'\left(1 - \frac{\alpha'}{2}\right) = \alpha\left(1 + \frac{\alpha}{2}\right)\left(1 - \frac{\alpha}{2} - \frac{\alpha^2}{4}\right) \approx \alpha \qquad (36)$$

in the first order. So, allowing totally varying free path lengths, the RT value of the Eyring formula converges against the Sabine value.

7 ANALYTICAL APPROACHES FOR PARTIALLY DIFFUSE SOUND FIELDS

In the following, some concepts shall be discussed which do not any longer assume homogeneous and/or isotropic sound fields, yet still diffuse reflections. As mentioned, even overall diffuse reflections do not guarantee a diffuse sound field, the irradiation strengths on the surfaces B_i may not be constant. A base for the next derivations is Kuttruff's integral equation for the irradiation strength $B(\boldsymbol{r}, t)$ [1] here reproduced in the time dependent case without (with switched-off) sound source:

$$B(\boldsymbol{r}, t) = \int_{S'} \rho(\boldsymbol{r}')B(\boldsymbol{r}', t - R/c)\frac{cos(\vartheta)cos(\vartheta')}{\pi R'^2}dS' \qquad (37)$$

where r is the receiver position, r´ the position of a radiating surface element dS, ϑ the incident angle, ϑ' the emission angle from dS´, $R = |\boldsymbol{r}' - \boldsymbol{r}|$ the distance between source and receiver position and $t - R/c$ the earlier time of emission from r´ (Fig. 9). The term $\frac{cos(\vartheta')}{\pi}$ is the probability density per solid angle due to the Lambert law, the term $cos(\vartheta)$ is due to the projection onto the receiving surface element on the other side. $\rho(\boldsymbol{r}')$ is the local reflection coefficient. The equation (already found by Clausius [6] for heat transfer) describes the radiation balance in a closed room with diffusely reflecting surfaces.

This integral equation can only be solved numerically by the time dependent 'radiosity' method. A compromise is an iteration with the assumption of an approximately exponential decay where the reverberation time is delivered as an Eigenvalue [8]. (This pays especially for solving the problem of computing the absorption degrees of specimen in the non-diffuse sound fields of reverberation rooms.)

7.1. Kuttruff's formula regarding the spreading of the absorption degrees

Aiming at an analytical approximation formula for a single exponential decay, Kuttruff found a formula taking into account the variance of the absorption degrees. All irradiation strengths B are assumed to decay exponentially and all distances R in Eqn. 37 are replaced by the same mean free path length Λ. By integrating the resulting equation over the whole surface S, and changing for practical reasons to a discrete formulation with N plane surfaces S_n, an 'effective' new absorption exponent is obtained:

$$\alpha'' = -ln\left(\frac{\sum_{n=1}^{N}\rho_n B_n S_n}{\sum_{n=1}^{N} B_n S_n}\right) \qquad (38).$$

where the ρ_n are the surface's reflection degrees, $\bar{\rho} = \sum_{n=1}^{N} \rho_n S_n / S$ is the mean reflection degree and the B_n are the (unknown) irradiation strengths, each assumed to be constant over a surface S_n. For constant irradiation strengths B, this would reproduce the Eyring absorption exponent $\alpha' = -ln\rho = -\ln(1-\alpha_m)$. Obviously, the new absorption exponent is not only a surface weighted but also a irradiation weighted average.

Especially interesting in praxis are rectangular rooms with high absorption on the (largest area) floor and low absorption elsewhere. For the simplified cases of total floor absorption and zero rest absorption, Kuttruff computed effective absorption exponents in the order of 10% (for a 1:1:1 cube) up to 20% (for a 5:2:1 shoe-box , where 5:2 is the proportion of the floor rectangle) over the Eyring value, hence, shorter reverberation times [1].

In the typical case of one receiving high absorptive floor and rather reflecting other surfaces, the reasonable guess for the unknown irradiation strengths B_n is that they are simply proportional to the other surfaces times their reflection coefficients. After inserting the respective equation into Eqn 38, and some approximation for small correction factors, the result for an effective new absorption degree taking a non-uniform absorption into account turns out to be

$$\alpha'' = \alpha' + \frac{\sum_{n=1}^{N}(\alpha_m - \alpha_n)(1-\alpha_n)S_n^2}{(1-\alpha_m)^2 \cdot S^2} \quad (39)$$

where the α_n are the absorption degrees of the surfaces, α_m their average, and α' is the Eyring absorption exponent. Analogously to Eyring is $\tau_{Kutt} = \Lambda/(c\,\alpha'')$ and $T_{Kutt} = 6\ln(10)\,\tau_{Kutt}$.

At least for the frequent cases of one dominating absorbing surface (usually the floor), the term to the right in Eqn. 39 is positive, and the Kuttruff formula yields lower RTs than according even the Eyring value: $T_{Sab} > T_{Ey} > T_{Kutt}$.

7.2. Fitzroy's subdivision into separate reverberation processes in x-, y- and z-direction

Another early approach to take the special effects of a non-uniform absorption distribution or, better to say, anisotropy into account, is that of Fitzroy [9]. Often, especially in planar rectangular rooms with few scattering and hence mixing, more or less separate reverberation processes establish in the three main axes' directions. Assigning for each direction *(x,y,z)* a specific mean absorption coefficient α_1, α_2 , α_3 and 'typical' mean free path lengths Λ_1, Λ_2 , Λ_3, one could derive, in the same way as for the Eyring RT (section 4.1), specific reverberation times T_1, T_2 and T_3 respectively. Fitzroy's empirical (somewhat naive) compromise is just to take the arithmetic surface weighted average of all three:

$$T_{Fr} = \frac{S_1}{S} \cdot T_1 + \frac{S_2}{S} \cdot T_2 + \frac{S_3}{S} \cdot T_3 \quad (40)$$

where the S_1, S_2 and S_3 are the surfaces of the room perpendicular to the x,y,z-direction. But: what are the values of the surfaces S_1, S_2 and S_3 in cases of non-rectangular rooms, the projected fractions of all surfaces into the three directions? The choice of such three orthogonal directions seems quite arbitrary.

7.3. Arau's improved reverberation formula

The same unanswered questions apply to the model of Arau [10] who further developed Fitzroy's model. The basic approach of his formula is to account for different classes of reflections ($N_i; i = 1,2,3$ for x,y,z; out of $N = \sum N_i$) such that (instead of just one stepwise decay as with the Eyring theory in Eqns. 14,15) the energy decay is (in the first approach) now described by

$$E(N) = E_0 \prod (1-\alpha_i)^{N_i} = E_0 \cdot e^{-\sum N_i \alpha_i'} \quad (41)$$

The first idea is that (as in a diffuse sound field, i.e. with constant surface irradiation) the probabilities to hit a surface S_i are $p_i = S_i/S$, such that the reflection numbers are $N_i = N \cdot p_i$. Then $\sum N_i \alpha_i' = N \cdot \sum p_i \alpha_i'$ and with

$$\overline{\alpha'} = \sum p_i \alpha_i' \quad (42)$$

a 'surface weighted absorption exponent' could be defined (and from that as with Eqns.(16,17) a reverberation time). This would be different from $\alpha_m' = -\ln(1-\alpha_m)$ (Eqn.15) and become infinite (and the RT zero) if just one $\alpha_i' = -ln(1-\alpha_i\ )$ would become infinite, if $\alpha_i \rightarrow 1$ (one 'open window'). So this approach (once formulated by Mellington – Sette as an 'improvement to Eyring' [1]) is definitely wrong. The reason is, as Kuttruff showed, that the p_i are just probabilities, but the reflection numbers are not exactly N_i but the probability for e.g. N_1 reflections obeys a binominal distribution

$$p(N_1) = \binom{N}{N_1} \cdot {q_1}^{N_1} \cdot {q_2}^{N_2} \text{ (with } N_2 = N - N_1\text{).} \quad (43)$$

There are many sequences of reflections of different classes. These are described by an respective binominal distributed sum over different decays (exponential functions as in Eqn. 41). Kuttruff's result is: the surface weighted averaging of a mean absorption degree α_m (Eqn. 6) as for the Sabine and Eyring formula is justified.

Now, also Arau went this way of introducing a binominal distribution (where a transition to a logarithmic normal distribution is not quite clear). The arithmetic average of the absorption exponent as in Eqn. 42 is now just applied to surfaces within the same class (e.g. opposite parallel walls) for the ***sequential*** effects. For the ***simultaneous*** reverberation processes, he derives the formula:

$$\overline{\alpha_{Arau}'} = \prod_{i=1}^{3} (\overline{\alpha_i'})^{p_i} \quad (44)$$

- an area weighted geometric mean of the weighted absorption in the x,y,z- directions. Component wise insertion into the usual reverberation time formulae (like Eqns. 16,17) yields for the reverberation time finally also an area weighted geometric mean:

$$T_{Arau} = {T_x}^{S_x/S} \cdot {T_y}^{S_y/S} \cdot {T_z}^{S_z/S} \quad (45)$$

The computed results, obtained separately for early and late RT, agreed astonishingly well – and much better than Sabine and Eyring – with measurements – at least in a rectangular and highly diffusing hall.

7.4. An Anisotropic Reverberation Model (ARM)

However, there is an important deficit at this and Fitzroy's method: the decisive 'mixing' between the different reverberation classes caused by more or less scattering walls is not taken into account. This exactly is possible with ARM [11]. The semi-analytical ARM takes absorption and scattering coefficients into account as well as the orientation of the surfaces, however, not their positions. As a compromise between simple analytic (often wrong) formulae and costly ray tracing methods, it assumes still a homogeneous but anisotropic sound field. The idea is to consider flowing sound energies in a group (typically some thousands) of angular ranges (like pyramidal beams but without defined origins) and to define coefficients describing transitions between them over the relevant surfaces depending on their absorption and scattering coefficients and their orientation. This leads to a linear system of ordinary differential equations. This system can be solved either by iteration or with Eigenvalues (partial RTs) and Eigenvectors (energy distributions). The result gives information on early and late reverberation times.

7 CONCLUSION

The "diffuse sound field" is a very idealistic assumption. The reasons for the difference between the Sabine and the Eyring formula are different tacit additional assumptions. The 'true' reverberation times lie between the Sabine value and the Eyring value, as described by the Kuttruff formula– provided the absorption degrees are small and not too different and all reflections are diffuse and totally mixing – which is an utopia. So actually, both the Sabine and the Eyring formula are wrong. Strictly speaking, they must not be applied in many cases of non-perfectly diffuse reflections i.e. in many realistic cases. Also all other reverberation formulae assume perfectly diffuse reflections – which is approximately often, but never perfectly the case. They yield rather small corrections depending on the distribution of the absorption. However, they cannot explain effects of partly geometrical reflections which may be dominating in cases of focusing on reflecting or absorbing surfaces causing e.g. flutter echoes in shoe-box-rooms or focusing effects in domes [12]. Reverberation times in non-diffuse sound fields depend on the room shape, the distribution of the absorption and especially the scattering coefficients. In two other papers, presented to this conference, some semi-analytical procedures are presented to compute reverberation times as a function of the scattering coefficients, taking the anisotropy into account [11]. For a semi-circular room with its focusing effects an analytical formula is presented [3].

6 REFERENCES

1. Kuttruff, H.: Room Acoustics, Elsevier Science Publishers Ltd., Barking, England
2. Joyce, W.B.; Sabine's reverberation time and ergodic auditoriums; J.Acoust.Soc.Am.58,3, (1975), 643-655
3. Stephenson, U.M.: Are there simple reverberation time formulae also for partially diffusely reflecting surfaces ?; in: proc. of INTERNOISE, New York, 2012
4. Vorländer, Michael: Auralization; Fundamentals of Acoustics, Modelling, Simulation, Algorithms and Acoustic Virtual Reality, 1. Ed. Springer-Verlag 2008
5. Kosten,C.W.; The mean free path in room acoustics; Acustica 10 [1960], 245
6. Clausius, R.; Die kinetische Theorie der Gase, 2. ed., (Vieweg, Braunschweig 1889), 51
7. Cremer, L., Müller, H.; Principles and Applications of Room Acoustics, Vol. 1; translated by T.J. Schultz; Applied Science Publishers, London, New York 1978
8. Kuttruff, H.; A Simple Iteration Scheme for the Computation of Decay Constants in Enclosures with Diffusely Reflecting Boundaries; J. Acoust.Soc. Am. 98 (1), 1995, 288
9. Fitzroy, D; A reverberation formula which seems to be more accurate with a non-uniform distribution of absorption; J. Acoust. Soc. Amer. 31 [1959], p. 893
10. Arau-Puchades, H.; An improved reverberation formula; Acustica 65 [1988], 163-180
11. Drechsler, S; The Anisotropic Reverberation Model (ARM) - a simplified method to calculate the sound energy decay and derived properties; in: proc. of INTERNOISE, New York, 2012
12. Stephenson, U.M; On the influence of the ceiling and audience profile on the reverberation time and other room acoustical parameters; in: proc. of Intern. Conf. on Auditorium Acoustics 2008 Oslo; Inst. of Acoustics, University of Salford, UK, 2008

Table 1 – Conditions for the diffuse sound field (in brackets:

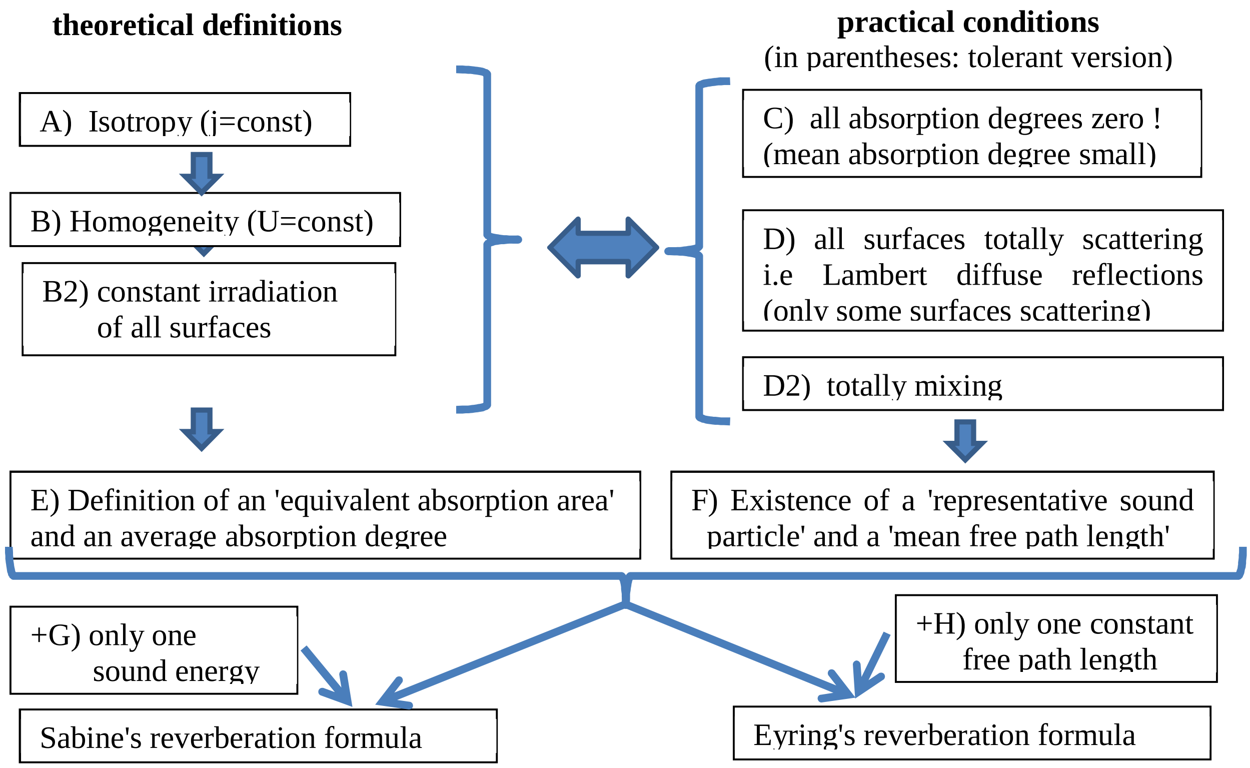

Fig. 1: Isotropy and homogeneity (same arrow lengths in every direction everywhere, at every 'array-cluster'); Fig. 1b: constant irradiation of all surfaces (B2) following from B) (homogeinity)

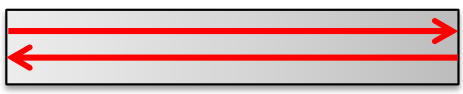

Fig 2 : Homogenous but anisotropic sound distribution : flutter echoes in a long rectangular room with reflecting front-end but absorbing side walls

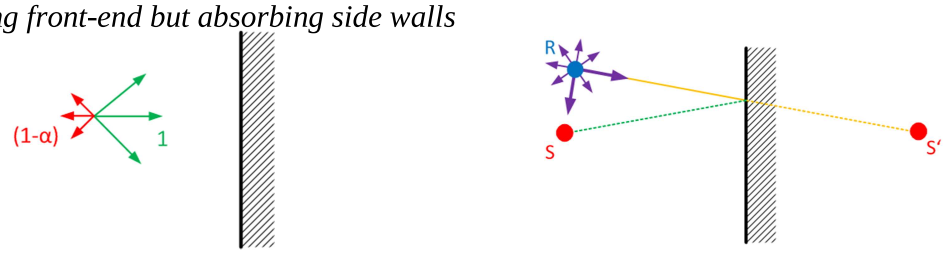

Fig. 3a: An absorbing piece of surface destroys isotropy,
3b): Each specular reflection destroys also isotropy (causes a peak in the array-cluster)

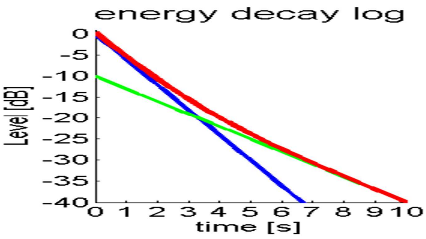

Fig. 4: Two different exponential energy decays (green and blue, logarithmically displayed) in weakly couples sub-spaces or intodifferent directions ; red curve: their sum – a non-.exponential decay

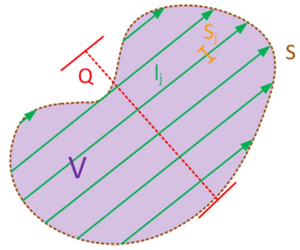

Fig. 6: A volume V of cross section Q subdivided into n 'channels' of cross section q and lengths l_i: $q \sum_{i=1}^{n} l_i = V$

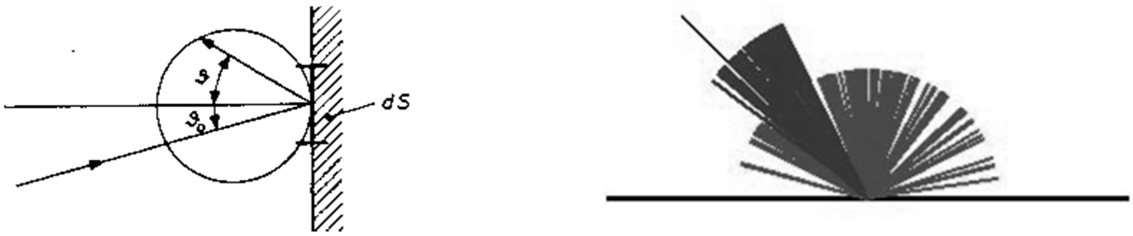

Fig. 5a: The Lambert directivity (cosine-function); 5b) reflected ray distributions simulated by drawing random numbers for scattering coefficients of 0, 0.25 and 1.

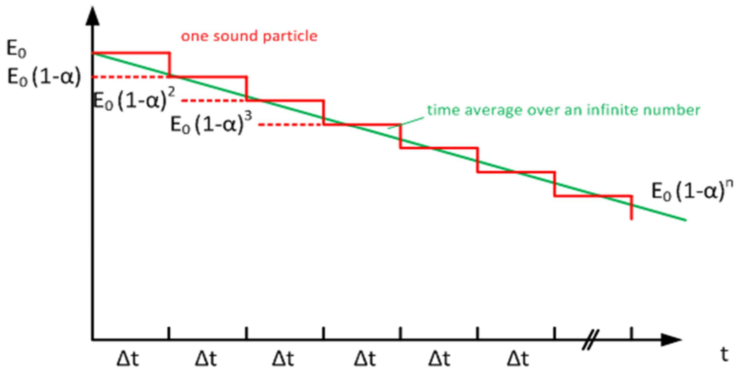

Fig. 7: The stepwise exponential energy decay as assumed by the Eyring theory as a function of the number of reflections N in constant time intervals Δt (energies in logarithmic scale); straight green line: time average over many sound particles

Fig. 8: If SP1 hits wall S1,the total energy E in the room is reduced. Tacitly assumed by Sabine, this information is transmitted immediately to SP2 such that its energy is also reduced. Then its future energy loss due to absorption on wall S2 will be smaller than otherwise leading to a longer reverberation time than due to Eyring.

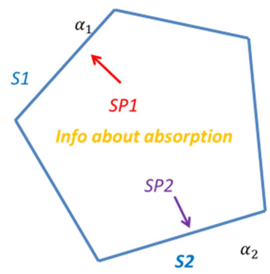

Fig. 9 Mutual irradiation of two pieces of surface dS and dS' in a room: illustration of the quantities in Kuttruffs integral equation [1]