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An Improved Reverberation Formula

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Summary

A new formula of the reverberation time is deduced taking into account the nonuniform distribution of absorption case in rooms. The formula is confirmed by an experimental analysis, that we present on the graphical recordings comparing our results with those of Eyring and Fitzroy.

Also we introduce in this paper a study of the effects of area and location of the absorbing material in a room. The results obtained with the new formula are in good agreement with the experimental measurements of Young, Knudsen and others.

Eine verbesserte Nachhallformel

Zusammenfassung

Es wird eine neue Formel für die Nachhallzeit abgeleitet, welche die ungleichförmige Verteilung der Absorption in Räumen berücksichtigt. Die Formel wird durch eine experimentelle Untersuchung bestätigt in der unsere Ergebnisse mit denen von Eyring und Fitzroy verglichen werden.

In dieser Arbeit werden weiterhin die Einflüsse der Fläche und Lage absorbierender Materialien im Raum untersucht. Die Ergebnisse, die mit der neuen Formel erhalten werden, stehen in guter Übereinstimmung mit Meßdaten von Young, Knudsen und anderen.

Une formule améliorée pour la réverbération

Sommaire

On présente une formule nouvelle donnant la durée de la réverbération du son dans une salle garnie d'une répartition non-uniforme de panneaux absorbants. La validité de la formule est controlée par une analyse expérimentale traduite en enregistrements graphiques où ses résultats sont comparés avec ceux des formules d'Eyring et de Fitzroy.

Dans le même article on développe une étude des effets de la situation et des dimensions des matériaux absorbants disposés dans la salle sur la durée de la réverbération. Les prévisions de la nouvelle formule se trouvent en bon accord avec les résultats expérimentaux de divers auteurs, et notamment avec ceux de Young et de Knudsen.

1. Introduction

We have ever admired for many years ago the empirical Fitzroy's equation [1], by its simplicity. It has been comprensible that important acousticians [2–4], included this formula in their books trying to explain the nonuniform distribution of absorption case in relation to the sound decays process and the reverberation time of rooms. We recall that Fitzroy proposed that the Eyring equation should be modified to consider the possibility that the sound field decays by a conjunction of three simultaneous Eyring-type decays between the pairs of parallel boundaries of the enclosure.

The empirical formula proposed by Fitzroy considers the reverberation time of room behaves like an area-weighted arithmetical mean of the reverberation periods in three directions.

We agree with the basic intuitive idea of Fitzroy, but our demonstration realized in section 2 results into another very different equation that we consider better related to the real sound decays in rooms.

2. Theoretical approach

When the walls of a room have a nonuniform distribution of absorption, it is not possible, after one has shut off the sound source, such as do Norris-Eyring for uniform distribution, to assume that all sound rays lose the same amount of energy in identical interval of time.

The principal acoustical quantity, of interest in this decay process, is the total amount of energy that remains in the room and its variation with time, given by the following equation:

$$D = \frac{10\lg\left(\dfrac{E_0}{E}\right)}{t}, \tag{1}$$

where D is the rate decay in dB per unit time, t is the time interval of the sound decay, E is the remainder energy, E_0 is the initial energy.

It is well known, that at each sound reflection on the walls of the room, the average sound energy density level, and therefore also the sound pressure level, experience a loss equal to: $-10\lg(1-\bar{\alpha})$, where $\bar{\alpha}$ is the mean energy absorption coefficient of the boundaries of the room.

Thus if the average number of reflections per unit time is N, the rate of decreasing sound pressure level will be:

$$D = -10\,N\,\lg(1-\bar{\alpha})\,. \tag{2}$$

Calling $\bar{a}$ the term $-\ln(1-\bar{\alpha})$:

$$\bar{a} = -\ln(1-\bar{\alpha})\,. \tag{3}$$

We deduce the following equation:

$$\bar{a} = \frac{D}{10}\,\frac{1}{N}\,\frac{1}{\lg \mathrm{e}}\,, \tag{4}$$

which allow me to describe $\bar{a}$ as mean decay rate absorption coefficient of the room, associated to $\bar{\alpha}$ by means of the eq. (3).

Also the coefficient $\bar{a}$ can be looked at as a Sabine coefficient [5].

The Sabine coefficient may exceed unit, this reason is inherent in its definition (3), and therefore is preferable state it as a pure number connected to the decay sound process.

From eqs. (1) and (4) it is possible to demonstrate the following expression:

$$E = E_0\,\mathrm{e}^{-Nt\bar{a}}\,. \tag{5}$$

Knowing that the sound decay results from multiple simultaneous and sequential reflections on the surfaces of the room, admiting that sequentials sound reflections occurs with preference between pairs of parallel walls and that the simultaneous sound reflections are produced in adjacent perpendicular walls, we establish the following coefficient $\bar{a}$ in three directions between parallel walls:

$$\bar{a} = (\bar{a}_x)^{x/S}\,(\bar{a}_y)^{y/S}\,(\bar{a}_z)^{z/S}\,, \tag{6}$$

where $\bar{a}_x$ is the mean decay rate absorption coefficient in the direction 1: $\bar{a}_x = -\ln(1-\bar{\alpha}_x)$, $\bar{a}_y$ the mean decay rate absorption coefficient in the direction 2: $\bar{a}_y = -\ln(1-\bar{\alpha}_y)$, $\bar{a}_z$ the mean decay rate absorption coefficient in the direction 3: $\bar{a}_z = -\ln(1-\bar{\alpha}_z)$, $\bar{\alpha}_x$ average energy absorptivity in x areas, $\bar{\alpha}_y$ average energy absorptivity in y areas, $\bar{\alpha}_z$ average energy absorptivity in z areas, x the ceiling + floor area, y the area of side walls, and z the area of the bottom walls; consequently: $S = x + y + z$.

Eq. (6), expresses the mean decay rate absorption coefficient a of the enclosure by its area weighted geometric mean on each one of the directions of the room that relate the different pairs of parallel boundaries.

The two conditions in our hypothesis are preserved in this equation:

1) The sequentiality is assured through the arithmetic mean of energy absorption coefficient between each pair of parallel boundaries.
2) The simultaneously is kept by means of the area weighted geometric mean of the decay rate absorption on each direction considered.

On the other hand, we see that the decay rate absorption coefficient $\bar{a}$ given by formula (6), such as an area weighted-geometric mean, is almost coincident to an area weighted – arithmetic mean coefficient, that for a near values of $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$, or quasi-uniform distribution of absorbent material, we have:

$$\bar{a} = \bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S} \simeq \frac{x}{S}\,\bar{a}_x + \frac{y}{S}\,\bar{a}_y + \frac{z}{S}\,\bar{a}_z\,.$$

It means that for a uniform distribution of the absorbing material, the two means will must be equals.

Also for small values of the energy absorption coefficients (diffuse sound field), we obtain the following expression approximated:

$$\frac{x}{S}\,\bar{a}_x + \frac{y}{S}\,\bar{a}_y + \frac{z}{S}\,\bar{a}_z \cong \frac{x}{S}\,\bar{\alpha}_x + \frac{y}{S}\,\bar{\alpha}_y + \frac{z}{S}\,\bar{\alpha}_z \quad (\equiv \bar{\alpha}_{\mathrm{av}})\,.$$

However for a non-uniform distribution of the absorbing material in general we will the following unequality:

$$\bar{a} = \bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S} \neq \frac{x}{S}\,\bar{a}_x + \frac{y}{S}\,\bar{a}_y + \frac{z}{S}\,\bar{a}_z\,.$$

For this case, associated to $\bar{a}$ of the eq. (6) there is an $\bar{\alpha}$, obtained through $\bar{a} = -\ln(1-\bar{\alpha})$, which is different to $\bar{\alpha}_{\mathrm{av}}$, where is also:

$$\left(\bar{\alpha}_{\mathrm{av}} = \frac{1}{S}\sum_i S_i\,\alpha_i\right).$$

The difference $(\bar{\alpha}_{\mathrm{av}} - \bar{a})$ shall give us a measure of the anisotropy and inhomogeneity of the sound field. However, if this difference is decreased, it shall means that the sound field also shall be reduced nearly to an homogeneous and isotropic field.

This non-homogeneous and anisotropic pattern of the sound field established on the room, which is not possible to explain with the traditional statistical theories, it is produced by an uneven distribution of the sound decay rate reflected from each wall of the enclosure, after one has shut off the sound source, due there is a sound rays lose in unequal amounts of energy to equality of interval of time on its boundaries.

By it is obvious that only the area weighted-geometric mean of the decay absorption coefficients, given by the formula (6), which are proportionals to the decay rates of the sound field, will enable us attain the symmetrization of the decay distribution values of the sound field non-homogeneous, approximating it to a normal statistical distribution. In section 3 we shall give a theoretical justification of it.

This area weighted geometric mean is especially valid for this statistical purpose and not happens the same with the classical area weighted arithmetic mean formulated before by other investigators in similar form.

Hence in summary my new proposition:

$$\bar{a} = \bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}\,.$$

It is a very powerful area weighted geometric mean that, in results computed, is coincident to traditional absorption's formules known for an homogeneous and isotropic sound field and hence to an uniform absorption distribution case.

But for a non-uniform distribution of the absorbing material it yields us two advantages:

1. A symmetrization of the decay distribution values of the non-homogeneous and anisotropic sound field.
2. An approximation of this sound field to a normal statistical distribution behavior.

Generalizing (6) we may write:

$$\bar{a} = \bar{a}_{x1}^{x_1/S}\,\bar{a}_{x2}^{x_2/S}\ldots\bar{a}_{xn}^{x_n/S}\,, \tag{7}$$

where $\bar{a}_{xi}$ is the mean decay rate absorption coefficient of two perpendicular walls to one defined direction i, xi is the total area of the two walls and S is the whole area of the room.

Substituting (6) in (5) we have:

$$E = E_0 \exp\{-N\,t\,\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}\} \tag{8}$$

and taking logarithms:

$$-\ln\frac{E}{E_0} = N\,t\,\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}\,. \tag{9}$$

Applying logarithms again and making use of the identity

$$\frac{x}{S}+\frac{y}{S}+\frac{z}{S}=1$$

we can write:

$$\left(\frac{x}{S}+\frac{y}{S}+\frac{z}{S}\right)\ln\left(-\ln\frac{E}{E_0}\right) = \ln t + \left(\frac{x}{S}+\frac{y}{S}+\frac{z}{S}\right)\cdot\ln N + \ln(\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S})\,. \tag{10}$$

Applying the equality

$$(x+y+z)\ln A = \ln(A^x A^y A^z)\,, \tag{11}$$

we can write:

$$\ln\left[\left(-\ln\frac{E}{E_0}\right)^{x/S}\left(-\ln\frac{E}{E_0}\right)^{y/S}\left(-\ln\frac{E}{E_0}\right)^{z/S}\right] = \ln[t\,N^{x/S}\,N^{y/S}\,N^{z/S}\,\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}] \tag{12}$$

or

$$\left(-\ln\frac{E}{E_0}\right)^{x/S}\left(-\ln\frac{E}{E_0}\right)^{y/S}\left(-\ln\frac{E}{E_0}\right)^{z/S} = t\,N^{x/S}\,N^{y/S}\,N^{z/S}\,\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}\,. \tag{13}$$

From this equation we find:

$$t = \left[\frac{\left(-\ln\frac{E}{E_0}\right)}{\bar{a}_x N}\right]^{x/S}\left[\frac{\left(-\ln\frac{E}{E_0}\right)}{\bar{a}_y N}\right]^{y/S}\left[\frac{\left(-\ln\frac{E}{E_0}\right)}{\bar{a}_z N}\right]^{z/S}. \tag{14}$$

Now if $t = T$ corresponds to a sound energy drop equal to 60 dB, i.e. the reverberation time of room. $(E/E_0 = 10^{-6})$ we have:

$$T = \left[\frac{6}{\bar{a}_x N \lg e}\right]^{x/S}\left[\frac{6}{\bar{a}_y N \lg e}\right]^{y/S}\left[\frac{6}{\bar{a}_z N \lg e}\right]^{z/S}. \tag{15}$$

This formula can be expressed as:

$$T = T_x^{x/S}\,T_y^{y/S}\,T_z^{z/S}\,, \tag{16}$$

with $T_x = \dfrac{6}{\bar{a}_x N \lg e}$, $T_y = \dfrac{6}{\bar{a}_y N \lg e}$, $T_z = \dfrac{6}{\bar{a}_z N \lg e}$.

For a multiform regular room with n directions connecting pairs of parallel walls, substituting eq. (7) in (5) and employing the same mathematical process it is possible to find the general expression:

$$T = \prod_{i=1}^{n} T_{xi}^{x_i/S}\,, \tag{17}$$

in which $T_{xi} = \dfrac{6}{\bar{a}_{xi} N \lg e}$.

Considering now by simplicity that:

$$N = \frac{c}{l_m}\,,$$

where

$$l_m = \frac{4V}{S}\,,$$

is the classical mean free path, such as is accepted by many acousticians, [6, 8–10] versus others who think and prove that there is some dependence on room shape [11–15], we can write from eq. (16):

$$T=\left[\frac{0.161\,V}{-S\ln(1-\bar{\alpha}_x)}\right]^{x/S}\left[\frac{0.161\,V}{-S\ln(1-\bar{\alpha}_y)}\right]^{y/S}\cdot\left[\frac{0.161\,V}{-S\ln(1-\bar{\alpha}_z)}\right]^{z/S}. \tag{18}$$

In the general case for $l_m \neq 4V/S$ we have:

$$T=\left[\frac{6\,l_m(\lg e)^{-1}}{-c\ln(1-\bar{\alpha}_x)}\right]^{x/S}\left[\frac{6\,l_m(\lg e)^{-1}}{-c\ln(1-\bar{\alpha}_y)}\right]^{y/S}\cdot\left[\frac{6\,l_m(\lg e)^{-1}}{-c\ln(1-\bar{\alpha}_z)}\right]^{z/S}. \tag{19}$$

Both eqs. (18) and (19), express the reverberation time of room as the result of the area-weighted geometric mean of the reverberation periods in each one of the directions considered. This is a new approach to the acoustical problem of the decay rate of sound within a room with a nonuniform distribution of absorption.

Supposing now that

$$\bar{\alpha}_x=\bar{\alpha}_y=\bar{\alpha}_z\equiv\bar{\alpha}\,,$$

i.e. the absorption uniformly distributed in all walls of the room, and introducing it in eq. (18) we obtain the Eyring's formula:

$$T=\frac{0.161\,V}{-S\ln(1-\bar{\alpha})}. \tag{20}$$

Also for low absorption values of $\bar{\alpha}_x$, $\bar{\alpha}_y$, $\bar{\alpha}_z$, we can deduce from eq. (18):

$$T=\left[\frac{0.161\,V}{S\,\bar{\alpha}_x}\right]^{x/S}\left[\frac{0.161\,V}{S\,\bar{\alpha}_y}\right]^{y/S}\left[\frac{0.161\,V}{S\,\bar{\alpha}_z}\right]^{z/S}. \tag{21}$$

If now were $\bar{\alpha}_x=\bar{\alpha}_y=\bar{\alpha}_z\equiv\bar{\alpha}$, we would derive the Sabine's equation:

$$T=\frac{0.161\,V}{S\,\bar{\alpha}}. \tag{22}$$

Also both eqs. (18) and (21) must be observed respectively as a modified Eyring and Sabine equations, integrating in them the possibility that the sound field decays by a especial process of three simultaneous Eyring-type, or Sabine type, decays between the pairs of parallel boundaries.

We are convinced that the formula (18) can express with enough approximation the reverberation time value of a room with a sound field desrandomized and therefore anisotropic and nonhomogeneous, because of the different absorption of the walls.

Actually, in our demonstration, when the reverberation time is calculated on the basis of the number of times, that the energy is reflected during a 60 dB decay, we have it is strongly dependent of the product of the three simultaneous Eyring potential relations between the inverse of absorption decays coefficients and the percentages of areas on each direction considered. However, in this case the influence of the variation of the mean free path, nearly to 9%, because to the different dimensionals ratios of the room, as it was indicated by Allred and Newhose [11], concern now scarcely here. We will check in the four section the applicability of our formula (18) in several practical cases showing its fullness validity.

Clarifying something more deeply our simple analysis, is easy to demonstrate from the eq. (18) that the reverberation time can be written so:

$$T=\frac{0.161\,V}{S\,\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}}=\frac{0.161\,V}{S\,\bar{a}}. \tag{23}$$

That is an especial Sabine form, more general that the former equation (22), expressing it the reverberation time as related to the degree of variation of the decay absorption coefficient on various principal surfaces weighted according in proportion to their respective areas.

Hence the formula (23) is a general expression necessary for calculating the reverberation time of an enclosure in where exists a non-uniform absorption distribution of material on its boundaries, which is related also to the Eyring formula through the eq. (3) starting from the eq. (23):

$$T=\frac{0.161\,V}{-S\ln(1-\bar{\alpha})}. \tag{24}$$

Formula also formulated before in eq. (20), but in where now the average energy absorption coefficient ever must be computed from the eq. (6), being valid only $\bar{\alpha}=\bar{\alpha}_{av}$, as formulated by Eyring, for the uniform absorption distribution case in where exists an homogeneous and isotropic sound field inside a room. Resting so clear for us the general system of averaging absorption process, question which has been ever a very obscure subject, or as says [5], except for the tacit assumption that it be selected to yield the correct answer.

3. Theoretical justification of the absorption formula product and calculation of the three dimensional reverberation decay rate representation

In this section we shall develop a theoretical justification of the formula product $\bar{a}$ given in eq. (6):

$$\bar{a}=\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}\,.$$

After we shall can compute, for an asymmetrical absorption disposition on the room, the reverberation curved process of the decay as a superposition of a three dimensional reverberation, which governs the "early decay time" and a following two-dimensional reverberation, with a much longer reverberation time, corresponding respectively to a mean decay rate followed by a smallest decay rate, being therefore the sound energy-density splitted in three parts:

$$E(t) = E_1\,\mathrm{e}^{-KD_1t} + E_2\,\mathrm{e}^{-KD_2t} + E_3\,\mathrm{e}^{-KD_3t}, \quad (25)$$

where D_1 is the initial decay rate, D_2 is the mean decay rate and D_3 is the smallest decay rate.

In demonstrating the first part of the question formulated we can assume with Kuttruff [9], but only for each pair's parallel boundaries of the room, with the objective be assured a minimum of the sequentiality phenomena, that:

$$E_x(\bar{N}_{x1}) = E_{x0}(1-\alpha_{x1})^{\bar{N}_{x1}}(1-\alpha_{x2})^{\bar{N}_{x2}}, \quad (26)$$

where α_{x1} is the energy absorption coefficient of the surface S_{x1} (i.e. floor), and α_{x2} is the energy absorption coefficient of the surface S_{x2} (i.e. cailing); for a rectangular room should be $S_{x1} = S_{x2}$. Further $\bar{N}_{x1}$ are the number of the sound collisions above area's portion S_{x1}, and $\bar{N}_{x2}$ are the number of the sound collisions above portion S_{x2}. Let be also $\bar{N}_x = \bar{N}_{x1} + \bar{N}_{x2}$ and $x = S_{x1} + S_{x2}$.

We have also that the probability be produced $\bar{N}_{x1}$ collisions with wall of portion S_{x1} among a total number of reflections $\bar{N}_x$, on this direction 1, is:

$$W(\bar{N}_{x1}) = \binom{\bar{N}_x}{\bar{N}_{x1}}\left(\frac{S_{x1}}{x}\right)^{\bar{N}_{x1}}\left(\frac{S_{x2}}{x}\right)^{\bar{N}_x-\bar{N}_{x1}}. \quad (27)$$

From (26) and (27) is possible shown with Kuttruff:

$$E_x(t) = E_{x0}\left(1 - \frac{S_{x1}\alpha_{x1} + S_{x2}\alpha_{x2}}{x}\right)^{\bar{N}_x}.$$

From which equation we have:

$$E_x(t) = E_{x0}\,\mathrm{e}^{N_x t \ln(1-\bar{\alpha}_x)}, \quad (28)$$

where $\bar{\alpha}_x = (S_{x1}\alpha_{x1} + S_{x2}\alpha_{x2})/x$ and assuming that $\bar{N}_x = N_x t$.

Writing this last eq. (28) for the two another directions 2 and 3 recollecting all equations together, we have:

$$\begin{aligned} E_x(t) &= E_{x0}\,\mathrm{e}^{-N_x t \bar{a}_x}, \\ E_y(t) &= E_{y0}\,\mathrm{e}^{-N_y t \bar{a}_y}, \\ E_z(t) &= E_{z0}\,\mathrm{e}^{-N_z t \bar{a}_z}, \end{aligned} \quad (29)$$

where

$$\begin{aligned} \bar{a}_x &= -\ln(1-\bar{\alpha}_x); \\ \bar{a}_y &= -\ln(1-\bar{\alpha}_y); \\ \bar{a}_z &= -\ln(1-\bar{\alpha}_z). \end{aligned}$$

Writing now the total energy $E(t)$ let be so such that:

$$E(t) = E_0\,\mathrm{e}^{-Nt\bar{a}} = E_0\,\mathrm{e}^{-N_x t \bar{a}_x}\,\mathrm{e}^{-N_y t \bar{a}_y}\,\mathrm{e}^{-N_z t \bar{a}_z}. \quad (30)$$

In the real domain of $\bar{a}_k$ values this equation gives the mean energy of a sound ray with hits the x walls N_x times in time t, the y-walls N_y times etc.

Recognizing here that $E(t)$ let be a value depending of three random variables $E_x(t)$, $E_y(t)$, $E_z(t)$; being also $N = N_x + N_y + N_z$; $S = x + y + z$; $\frac{N_x}{N} = \frac{x}{S}$; $\frac{N_y}{N} = \frac{y}{S}$; $\frac{N_z}{N} = \frac{z}{S}$; and a the decay absorption coefficient mean.

We look that defining $E(t)$ we have a distribution of values $E_x(t)$, $E_y(t)$, $E_z(t)$, for each distribution of values $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$, than in general are very different among them because that $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$, are also very un-equals for an asymmetrical absorption disposition on the room.

Considering now other random distribution of $E(t)$, through $E_x(t)$, $E_y(t)$, $E_z(t)$, replacing in exponents $\bar{a}_i$ by $\lg\bar{a}_i$, this new distribution is a logarithm-normal distribution, we have:

$$\begin{aligned} \bar{E}(t) &= E_0\,\mathrm{e}^{-Nt\lg\bar{a}} \\ &= E_0\,\mathrm{e}^{-N_x t \lg\bar{a}_x}\,\mathrm{e}^{-N_y t \lg\bar{a}_y}\,\mathrm{e}^{-N_z t \lg\bar{a}_z}. \end{aligned} \quad (31)$$

Comparing exponents we have:

$$\lg\bar{a} = \frac{N_x}{N}\lg\bar{a}_x + \frac{N_y}{N}\lg\bar{a}_y + \frac{N_z}{N}\lg\bar{a}_z$$

being hence derived:

$$\bar{a} = \bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}. \quad (32)$$

For an uniform or quasi-uniform absorption disposition in where $E_x(t)$, $E_y(t)$, $E_z(t)$ are very similar because that $\bar{a}_x \simeq \bar{a}_y \simeq \bar{a}_z$, is not necessary in this case to make a symmetrization of values taking logarithms above $\bar{a}_i$; then comparing exponents in eq. (30) we find:

$$\bar{a} = \frac{x}{S}\bar{a}_x + \frac{y}{S}\bar{a}_y + \frac{z}{S}\bar{a}_z. \quad (33)$$

In our discourse we have assumed with Kuttruff, a binomial distribution of the sound collisions on each direction, which presents a normal statistical tendency, being only weighted with this procedure the asymmetry due to the area's proportion.

It is beautiful and good that the sound collisions present a normal distribution tendency, and it is logical and assured when the absorbing material is distributed in uniform form on the walls of the room.

However, when the absorbing material is placed in asymmetrical form on the room, also shall be asymmetrical or unequal the different sound decay rates reflected from each wall of the room.

We ask you then: How is possible to connect the normal distribution behavior of the sound collisions, formulated by Kuttruff, with the sound decay rates asymmetry produced by the absorption of the different materials placed on the walls?

We think that Kuttruff had reason above the sound collisions but we look also is necessary to get that the sound reflected, after the absorption on the wall, presents a normal distribution tendency. It has been easy for us formulating the area weighted geometric mean above the mean decay rate absorption coefficients $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$, that we know than also are proportionals to the sound decay rates on each direction.

With it we are transforming the values $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$ such as a logarithm-normal distribution: $\lg \bar{a}_x$, $\lg \bar{a}_y$, $\lg \bar{a}_z$. Where its arithmetical weighted mean is:

$$\lg \bar{a} = \frac{x \lg \bar{a}_x + y \lg \bar{a}_y + z \lg \bar{a}_z}{S}.$$

From which equation is deduced:

$$\bar{a} = \bar{a}_x^{x/S}\, \bar{a}_y^{y/S}\, \bar{a}_z^{z/S}.$$

This a is the mean of the log-distribution. And also the factor dispersion d is:

$$d = \operatorname{anti\,log} \left\{ \frac{x}{S} (\lg \bar{a}_x)^2 + \frac{y}{S} (\lg \bar{a}_y)^2 + \frac{z}{S} (\lg \bar{a}_z)^2 - \left[\frac{x}{S} \lg \bar{a}_x + \frac{y}{S} \lg \bar{a}_y + \frac{z}{S} \lg \bar{a}_z \right]^2 \right\}^{1/2}. \tag{34}$$

Being its central mass (68%) ranging among:

$$\bar{a}_i = \bar{a}\,(\text{factor dispersion})^{+1} \text{ and}$$
$$\bar{a}_f = \bar{a}\,(\text{factor dispersion})^{-1}. \tag{35}$$

This central mass express the interval among a maximum and minimum values of the log-distribution in where both are yet typical values of the distribution.

It is known [25], that the attainment of a logarithm-normal distribution can be produced by the conjunction of several random variables related by multiplication, meaning it also that the variations effect of a random variable is proportional to the existent value of it in this time.

But we see it is confirmated as followed by Sabine's coefficient, starting from his energy balance principle:

$$\frac{\mathrm{d}E}{\mathrm{d}t} + k\,\bar{a}_s E = 0.$$

From which I obtain:

$$\bar{a}_s = -\frac{k'}{E} \frac{\mathrm{d}E}{\mathrm{d}t}.$$

This means that the instantaneous loss of energy is always a constant percentage of the instantaneous amount of energy. By another side inherent to the same definition of the energy balance principle of Sabine we have that $\bar{a}_s$ coefficient accounts only for reflections occurring simultaneously but fails to allow for those that occur sequentially. However we recollect, inside of $\bar{a}_s$, the sequentially effect of the reflections through the arithmetic mean of energy absorption coefficients among each pair's boundaries, as formulated by Kuttruff, taking us only parallel surfaces and considering three principal directions; so we can write:

$$\bar{a}_x = -\ln(1 - \bar{\alpha}_x),$$
$$\bar{a}_y = -\ln(1 - \bar{\alpha}_y),$$
$$\bar{a}_z = -\ln(1 - \bar{\alpha}_z).$$

This is an oversimplification very useful recollecting all effects of simultaneity and sequentiality on minimum conditions required. Our assumption about $\bar{a}$ is complicated by its deep casiness, that only can be explained it by the logarithm-normal distribution above $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_z$ coefficients. With it we obtain the symmetrization of the values distribution and its approximation to a statistical normal behavior in coherence to the postulated by Kuttruff for $W(n)$ collisions or reflections. Do not made it should be not logical for our case of non-uniform disposition of the absorbing material. But is is not problem for uniform case in where the statistical normality of the absorption process is assured by definition.

Answering to the second question formulated by us in the beginning of this section above the eq. (25) showing the energy-density splitted in three parts, we have:

As showed by this theory we write:

$$T = \frac{0.161\, V}{S\,\bar{a}}, \tag{36}$$

where $\bar{a}$ is the area weighted geometrical mean (see eq. (6)). And also is:

$$D = \frac{60}{T}. \tag{37}$$

Hence from eqs. (35), (36) and (37) we have:

$$D_1 \equiv D_i = k\,\bar{a}_i, \tag{38.1}$$
$$D_2 \equiv \bar{D} = k\,\bar{a}, \tag{38.2}$$
$$D_3 \equiv D_f = k\,\bar{a}_f, \tag{38.3}$$

where D_i expresses the maximum typical decay-rate of the log-normal distribution, D_f is the smallest typical decay value of the distribution, and $\bar{D}$ is the mean decay of the same.

Therefore by introducing (38.1), (38.2) and (38.3) in the eq. (25) we obtain the total energy of the room:

$$E(t) = E_1\, \mathrm{e}^{-k\bar{D}dt} + E_2\, \mathrm{e}^{-k\bar{D}t} + E_3\, \mathrm{e}^{-k(\bar{D}/d)t}. \tag{39}$$

We also see from the eq. (34) that the factor dispersion d is equal to 1 ($d = 1$) for the uniform or quasi-uniform absorption case disposition in where $\bar{a}_x \simeq \bar{a}_y \simeq \bar{a}_z$.

For this case from (39) we find:

$$E(t) = E' \mathrm{e}^{-k \bar{D} t}$$

and therefore will exist an only slope $\bar{D}$ defining the absorption process. It is logical and known!

Analysing it with more detail we have:

For uniform or quasi-uniform disposition we must remember that:

$$\bar{a}_x^{x/S} \, \bar{a}_y^{y/S} \, \bar{a}_z^{z/S} = (x/S) \bar{a}_x + (y/S) \bar{a}_y + (z/S) \bar{a}_z$$

in where:

$$\bar{a}_x = -\ln\left(1 - \frac{S_{x1}\alpha_{x1} + S_{x2}\alpha_{x2}}{x}\right) = -\ln(1-\bar{\alpha}_x),$$

$$\bar{a}_y = -\ln\left(1 - \frac{S_{y1}\alpha_{y1} + S_{y2}\alpha_{y2}}{y}\right) = -\ln(1-\bar{\alpha}_y),$$

$$\bar{a}_z = -\ln\left(1 - \frac{S_{z1}\alpha_{z1} + S_{z2}\alpha_{z2}}{z}\right) = -\ln(1-\bar{\alpha}_z).$$

If $\bar{\alpha}_x = \bar{\alpha}_y = \bar{\alpha}_z \equiv \bar{\alpha}$ (uniform disposition), we have for this case:

$$\bar{a}_x^{x/S} \, \bar{a}_y^{y/S} \, \bar{a}_z^{z/S} = -\ln(1-\bar{\alpha})$$

being hence:

$$T = \frac{0.161\,V}{-S\ln(1-\bar{\alpha})}.$$

This is the Eyring formula and therefore $\bar{D} = (60/T)$ shall be an Eyring decay rate. For this case we remark that the factor dispersion is $d = 1$ and therefore only exist one slope.

However for a *non-uniform absorption disposition* we have three slopes on the decay rate curve:

First slope

$$D_i = \bar{D}\,d,$$

where $\bar{D}$ is the mean decay rate and d is the factor dispersion given by formula (34).

Second slope

$$\bar{D}\,(\text{mean decay}) = 60/T,$$

where $T = \dfrac{0.161\,V}{S\,\bar{a}_x^{x/S}\,\bar{a}_y^{y/S}\,\bar{a}_z^{z/S}}$.

Third slope

$$D_f = \bar{D}/d,$$

where $\bar{D}$ is the mean decay and d is the factor dispersion.

These first and third slopes are a logical consequence of our logarithm-normal distribution defined by the asymmetry of the coefficients $\bar{a}_x, \bar{a}_y, \bar{a}_z$.

Fig. 1. Three components of a reverberation decay.

Experimentally the decay rate $\bar{D}\,d$ can be confused with the initial sags of the decay curve, or also sometimes we can regard its slope confused as incorporated to the slope mean.

In the Fig. 1 we have represented schematically the decay-rate curved process.

Now we shall compute two particular cases:

First case

Let be $\bar{a}_x = 0.472$, $x/S = 0.326$,
$\bar{a}_y = 0.024$, $y/S = 0.412$,
$\bar{a}_z = 0.024$, $z/S = 0.261$.

This case was analyzed by Knudsen et al. and it is showed in the example 7) of section 5 of this paper.

For this case Knudsen experimentally measured:

$\bar{D} = 17.6$ dB/s, hence $T_2 = 3.409$ s,
$\bar{D}_f = 6.2$ dB/s, hence $T_3 = 9.67$ s.

We have computed by our procedure:

$D_i = 71.04$ dB/s, being $T_1 = 0.84$ s,
$\bar{D} = 17.55$ dB/s, hence $T_2 = 3.41$ s,
$D_f = \bar{D}/d = 4.33$ dB/s, hence $T_3 = 13.8$ s,

where $d = 4.048$ is the factor dispersion for this particular case and T_1, T_2, T_3 are the reverberation times associated to the decays.

Second case

I consider the same room with the following decay absorption coefficients:

$$\bar{a}_x = 0.472, \quad \bar{a}_y = 0.472, \quad \bar{a}_z = 0.024.$$

For this case $d = 3.44$ and $\bar{a} = 0.217$. Hence we have:

$D_i = 233.47$ dB/s, being $T_1 = 0.25$ s
(first slope: early decay rate),
$D = 67.87$ dB/s, being $T_2 = 0.884$ s
(second slope: mean),
$D_f = 19.7$ dB/s, being $T_3 = 3.04$ s
(third slope: smallest value end).

This values are a logical values for an asymmetrical absorption disposition on the room.

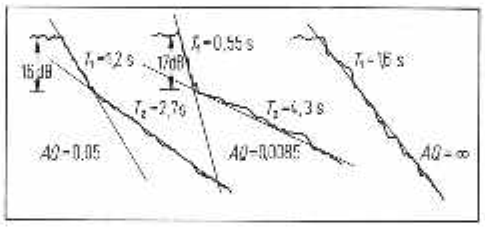

Fig. 2. Graphical recordings of ref. [23].

Following I present some graphical recordings registered by Bruel [23], see Fig. 2, showing the broken lines for a typical similar cases.

In conclusion our new theory keep all the following points:

1. For an uniform absorption distribution my formula is coincident to the classical theory of reverberation, recollecting in a one expression all the effects of Sabine and Eyring formules of the reverberation phenomena.
2. My formula for an asymmetrical absorption disposition of material on the room is in a good agreement with the mean experimental values obtained by several investigators. We shall look it in the next sections.
3. My concept of logarithm-normal distribution enable us also to calculate the smallest typical value of the decay-rate. Never computed its value by none theory anterior, but ever regarded in graphical recordings.
4. My theory is coherent with the normal tendency distribution of collisions formulated by Kuttruff, and also in coherence it also with formulated by Kosten and Clausius for the mean free path definition.
5. However, this formula is bad for extremal cases. You have reason about it. If for example $\bar{a}_x = 0$, $\bar{a}_x = -\ln(1-\bar{\alpha}_x)$, this should imply that be $\bar{\alpha}_x = 0$. Nevertheless this case does not exist in the practical reality because all materials have something absorption.

   The worse case is that, for example, let be $\bar{a}_x = \infty$ and $\bar{a}_y$, $\bar{a}_z$ finites. Then associated to $\bar{a}_x$ we have a $\bar{\alpha}_x = 1$ when $\bar{\alpha}_y, \bar{\alpha}_z < 1$, being hence $\bar{a} = \infty$ and $T = 0$. It is a very bad result but also is difficult that we have such so absorbent walls.

   We believe you must regard our formula applied for a typical examples of the usual reality, for example a floor very absorbent $\alpha_{x1} \cong 1$ with a ceiling very reflectant $\alpha_{x2} \cong 0$ being therefore in consequence $\bar{\alpha}_x < 1$. And so for similar distributions on the another surfaces of the room.

4. Practical cases and measurements

Twelve cases are studied in the present work. Between them only four examples were selected from acoustical references. Other cases have been collected from our own-self experience in measurements for acoustical designing of auditoriums, theaters and TV rooms. We have considered in this article only the 500 Hz frequency by reasons of simplicity.

On the Table I we present for all the rooms, the geometrical ratios, the percentages of area on each direction, the average energy absorption coefficients between pair parallel of boundary walls its particular associated decay absorption coefficient and also the average absorption energy coefficient of the whole boundaries.

We must remark that normally $\bar{\alpha}_{av}$ is different to the $\bar{\alpha}$ obtained from eq. (3) with $\bar{a}$ deduced by means eq. (6). Only for sound diffuse condition, or also for a uniform distribution of absorption, will be truth that $\bar{\alpha}_{av} = \bar{\alpha}$.

The five first rooms analyzed belong to Katalonien Television TV3, of the Generalitat of Katalonien, where we have worked in the acoustical design project. The case 1 is the called "Estudi Audio". This is a very especial room used for double purpose. The floor is moquette, the ceiling is composed by a metallic perforated system with fiber glass, of 30 kg/m^3 density, in its back zone with an air cavity of 80 cm of thickness.

In the front and rear walls are located 15 Resonators A1, 5 Resonators A2 and 5 Resonators A3.

The A1, A2, A3 resonators were calculated using Ingard-Bolt method [16] and were manufactured especially for this acoustical work.

The remaining zones of the bottom walls are of concrete painted.

The side walls of this room 1, there are 20 A1 resonators, 10 A2 resonators, 10 A3 resonators and the remainder is concrete painted.

The case 2 is the "Estudi 3" of TV3. The materials composition are described in reference [17].

The case 3 is the "Polivalent" study of TV3, where the floor is a especial rubber pavement.

The ceiling is a fine shaving of wood called Landa in commercial terms. The four meters, starting from floor level, in all the walls, is concrete painted.

In a lateral walls there are placed Ipakell modulus, material similar to egg box, in random form with air cavities of 5 cm and 25 cm. The composition of the rear wall is similar to the side wall described before. In the opposite side wall there is an especial foam called Soundcoat of 25 mm of thickness fixed.

The front wall is composed with Soundcoat foam material fixed on the wall surface.

Table I.
Dimensional ratios and absorption coefficients.

Case	Dimensions	Percentage area	Absorption coefficients		
			$\bar{\alpha}_i$ (energy)	$\bar{a}_i$ (decay)	$\bar{\alpha}_{av}$ (whole energy average)[1]
1	Ratio:				
	$a =$ 10,65 m	$\frac{x}{S} = 0.453$	$\bar{\alpha}_x = 0.306$	$\bar{a}_x = 0.365$	$\bar{\alpha}_{av} = 0.269$
	$b =$ 7.10 m				
	$c =$ 5.15 m	$\frac{y}{S} = 0.328$	$\bar{\alpha}_y = 0.252$	$\bar{a}_y = 0.289$	
	$V =$ 389.42				
	$S =$ 334.06	$\frac{z}{S} = 0.219$	$\bar{\alpha}_z = 0.213$	$\bar{a}_z = 0.246$	
2	Ratio:				
	$a =$ 22.20 m	$\frac{x}{S} = 0.439$	$\bar{\alpha}_x = 0.211$	$\bar{a}_x = 0.237$	$\bar{\alpha}_{av} = 0.489$
	$b =$ 17.90 m				
	$c =$ 12.84 m	$\frac{y}{S} = 0.310$	$\bar{\alpha}_y = 0.718$	$\bar{a}_y = 1.266$	
	$V =$ 5 022.88				
	$S =$ 1 808.49	$\frac{z}{S} = 0.250$	$\bar{\alpha}_z = 0.693$	$\bar{a}_z = 1.182$	
3	Ratio:				
	$a =$ 22.60 m	$\frac{x}{S} = 0.463$	$\bar{\alpha}_x = 0.315$	$\bar{a}_x = 0.378$	$\bar{\alpha}_{av} = 0.311$
	$b =$ 14.90 m				
	$c =$ 11,04 m	$\frac{y}{S} = 0.227$	$\bar{\alpha}_y = 0.303$	$\bar{a}_y = 0.361$	
	$V =$ 3 717				
	$S =$ 1 450	$\frac{z}{S} = 0.310$	$\bar{\alpha}_z = 0.312$	$\bar{a}_z = 0.374$	
4	Ratio:				
	$a =$ 14.80 m	$\frac{x}{S} = 0.420$	$\bar{\alpha}_x = 0.315$	$\bar{a}_x = 0.378$	$\bar{\alpha}_{av} = 0.158$
	$b =$ 10 m				
	$c =$ 8.23 m	$\frac{y}{S} = 0.233$	$\bar{\alpha}_y = 0.045$	$\bar{a}_y = 0.046$	
	$V =$ 1 218				
	$S =$ 704	$\frac{z}{S} = 0.345$	$\bar{\alpha}_z = 0.045$	$\bar{a}_z = 0.046$	
5	Ratio:				
	$a =$ 34.80 m	$\frac{x}{S} = 0.489$	$\bar{\alpha}_x = 0.212$	$\bar{a}_x = 0.238$	$\bar{\alpha}_{av} = 0.124$
	$b =$ 23.10 m				
	$c =$ 14.5 m	$\frac{y}{S} = 0.307$	$\bar{\alpha}_y = 0.040$	$\bar{a}_y = 0.0408$	
	$V =$ 11 656				
	$S =$ 3 286	$\frac{z}{S} = 0.204$	$\bar{\alpha}_z = 0.040$	$\bar{a}_z = 0.0408$	
6	Ratio:				
	$a =$ 5.73 m	$\frac{x}{S} = 0.473$	$\bar{\alpha}_x = 0.09$	$\bar{a}_x = 0.094$	$\bar{\alpha}_{av} = 0.074$
	$b =$ 5.58 m				
	$c =$ 3.15 m	$\frac{y}{S} = 0.260$	$\bar{\alpha}_y = 0.06$	$\bar{a}_y = 0.062$	
	$V =$ 100.71				
	$S =$ 135.19	$\frac{z}{S} = 0.267$	$\bar{\alpha}_z = 0.06$	$\bar{a}_z = 0.062$	
7	Ratio:				
	Non	$\frac{x}{S} = 0.640$	$\bar{\alpha}_x = 0.187$	$\bar{a}_x = 0.210$	$\bar{\alpha}_{av} = 0.259$
	rectangular				
	form	$\frac{y}{S} = 0.194$	$\bar{\alpha}_y = 0.368$	$\bar{a}_y = 0.459$	
	$V =$ 2 240				
	$S =$ 1 347	$\frac{z}{S} = 0.162$	$\bar{\alpha}_z = 0.415$	$\bar{a}_z = 0.536$	
8	Ratio:				
	Non	$\frac{x}{S} = 0.407$	$\bar{\alpha}_x = 0.250$	$\bar{a}_x = 0.287$	$\bar{\alpha}_{av} = 0.385$
	rectangular				
	form	$\frac{y}{S} = 0.296$	$\bar{\alpha}_y = 0.543$	$\bar{a}_y = 0.784$	
	$V =$ 10 352				
	$S =$ 2 438	$\frac{z}{S} = 0.297$	$\bar{\alpha}_z = 0.413$	$\bar{a}_z = 0.532$	

Table I. Continued.

Case	Dimensions	Percentage area	Absorption coefficients		
			$\bar{\alpha}_i$ (energy)	$\bar{a}_i$ (decay)	$\bar{\alpha}_{av}$ (whole energy average)[1]
9	Ratio:				
	Non rectangular	$\frac{x}{S} = 0.486$	$\bar{\alpha}_x = 0.368$	$\bar{a}_x = 0.4588$	$\bar{\alpha}_{av} = 0.264$
	form $V = 81\,000$	$\frac{y}{S} = 0.287$	$\bar{\alpha}_y = 0.176$	$\bar{a}_y = 0.194$	
	$S = 16\,471$	$\frac{z}{S} = 0.227$	$\bar{\alpha}_z = 0.153$	$\bar{a}_z = 0.166$	
10	Ratio:				
	Non rectangular	$\frac{x}{S} = 0.486$	$\bar{\alpha}_x = 0.084$	$\bar{a}_x = 0.0877$	$\bar{\alpha}_{av} = 0.126$
	form $V = 81\,000$	$\frac{y}{S} = 0.287$	$\bar{\alpha}_y = 0.176$	$\bar{a}_y = 0.194$	
	$S = 16\,471$	$\frac{z}{S} = 0.227$	$\bar{\alpha}_z = 0.153$	$\bar{a}_z = 0.166$	
11	Ratio:				
	Non rectangular	$\frac{x}{S} = 0.527$	$\bar{\alpha}_x = 0.410$	$\bar{a}_x = 0.528$	$\bar{\alpha}_{av} = 0.290$
	form $V = 87\,480$	$\frac{y}{S} = 0.286$	$\bar{\alpha}_y = 0.120$	$\bar{a}_y = 0.128$	
	$S = 15\,340$	$\frac{z}{S} = 0.187$	$\bar{\alpha}_z = 0.210$	$\bar{a}_z = 0.236$	
12	Ratio:				
	Non rectangular	$\frac{x}{S} = 0.527$	$\bar{\alpha}_x = 0.410$	$\bar{a}_x = 0.528$	$\bar{\alpha}_{av} = 0.403$
	form $V = 87\,480$	$\frac{y}{S} = 0.286$	$\bar{\alpha}_y = 0.450$	$\bar{a}_y = 0.600$	
	$S = 15\,340$	$\frac{z}{S} = 0.187$	$\bar{\alpha}_z = 0.310$	$\bar{a}_z = 0.370$	

[1] $\bar{\alpha}_{av} = \frac{1}{S}\sum S_i\,\alpha_i$ and also $\bar{\alpha}_{av} = \frac{x}{S}\bar{\alpha}_x + \frac{y}{S}\bar{\alpha}_y + \frac{z}{S}\bar{\alpha}_z$.

The absorption energy coefficient of the materials, were tested in our laboratory using the reverberant room method (ISO R 354).

The case 4 is the "Estudi de Noticies" of the TV3. Its description corresponds to the moment before beginning the acoustical treatment. Only in the ceiling there was a fine shaving of wood material. The remainders walls and floor were of concrete.

The case 5 is the "Estudi 1" of TV3. Its description obeys to a room without acoustical treatment, only the ceiling was treated with Landa material. An other surfaces were of concrete.

The case 6 is an empty of room concrete with 7.33 m^2 of absorptive material tested by us.

The case 7 is a hall, acoustically desgined by us, called Poliorama Theater. Its description corresponds to the final state after the acoustical treatment.

The architects were the Martorell-Bohigas-Mackay group. The composition of this hall can be found in reference [18]. The volume in cubic meters indicated in Table I is the real air volume.

In this case the amphitheater zone has been considered to belong to the rear wall assuming that is total surface is added to it.

The case 8 is the "Palau de la Musica Catalana of Barcelona". This hall is a very important one. This building is an historical monument of Katalonien, is the most famous hall of Spain intended for choral and symphony music. Designed by Lluis Domenech i Montaner, Architect, was builded in 1908, following the modernisme canons style.

In Figs. 3, 4 and 5 we present the interior of this hall.

Before the remodelation works, carried by the architect's group formed by Oscar Tusquets, and Lluis Clotet et al., the principal architectonic components of the hall were:

Floor: Upholstered chairs above plataform of wood.

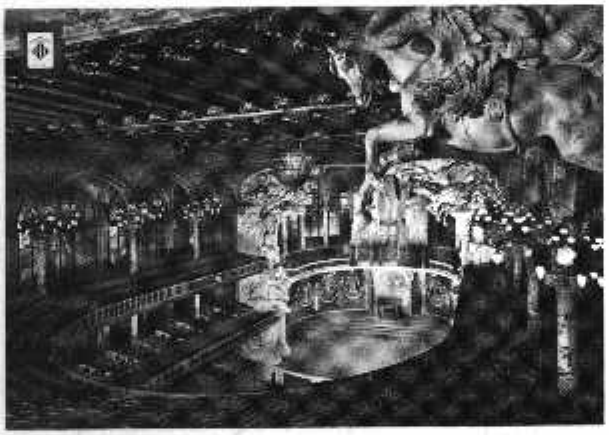

Fig. 3. Concerts drawing-room. General view.

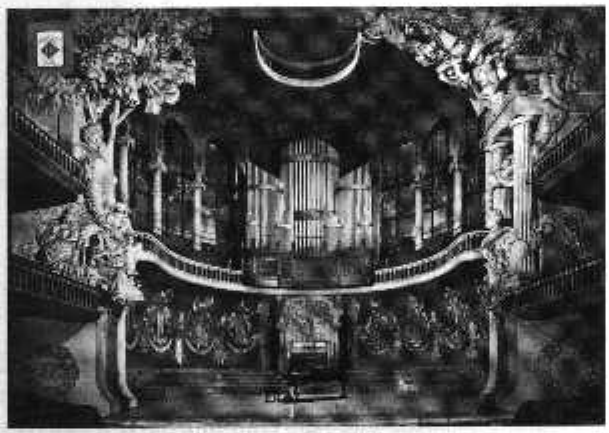

Fig. 4. General view of the semicircle.

Ceiling: Small mosaics of coloured ceramic, and in its center a large and wide chandlier incorporated inside of skylight zone, composed of the light glass leaded that in Germany, where they were manufactured in 1908, were called Orfeo glass.

Side walls: A great area of glass leaded.
Plataform of wood on floor of the lateral boxes plus upholstered chairs.
Remainder zones: Mosaic ceramic, stone and plaster.

Rear wall: In plant level: Stone painted and curtains.
Second level: Chairs above plataform of wood plus curtains.
Remainders regions: Mosaic ceramic, stone and plaster.
Third level: Chairs above plataform of wood. Glass leaded windows. Remainder zones: Mosaic ceramic, stone and plaster.

Frontal wall (scenary): Mosaic, glass leaded, organ, curtains, stone and plaster.

The real volume air is 10 352 m^3. The total number of upholstered chairs is 2100.

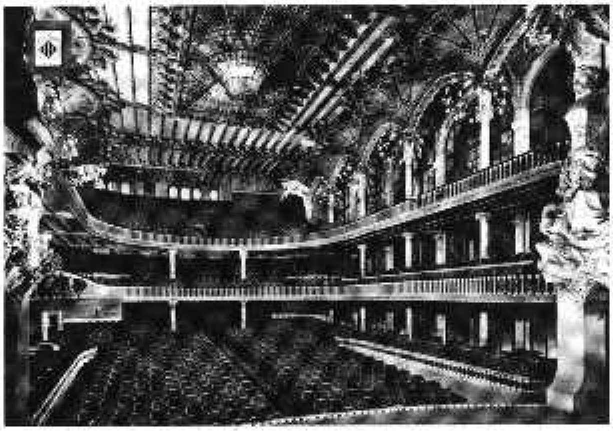

Fig. 5. Concerts drawing – hall from the stage.

In our study we have considered the area of amphitheaters floor as added to the real area of the rear wall.

On side walls we have assumed 800 chairs, in bottom walls we have considered 600 chairs, in plant we had 700. The energy absorption coefficient of chair was 0.24. The reverberation time was measured in an empty hall.

The cases 9 and 10 were deduced from reference [19] on the Cleveland Public Auditorium. The first case 9 corresponds to the auditorium after treatment, and the case 10 for the hall before acoustical treatment. In both cases the surfaces on where resting the audience has been considered forming part of the area of walls.

The cases 11 and 12 can been found in reference [20]. The first case treats the Rochester War Memorial Auditorium without audience, and the second considers the same hall with a 6000 persons attendance.

The Table II shows the values of the reverberation time obtained from eq. (18), using the values of

Table II.
Reverberation time using the values of Table I.

Case	Reverberation time			
	New formulae (18) s	Eyring s	Fitzroy s	Experimental s
1	0.606	0.599	0.613	0.66
2	0.75	0.66	1.02	0.84
3	1.122	1.108	1.121	1.16
4	2.49	1.62	3.89	2.4
5	5.90	4.32	8.32	6
6	1.58	1.55	1.68	1.57
7	0.95	0.90	1.02	0.96
8	1.44	1.37	1.57	1.47
9	2.80	2.61	3.13	–
10	6.26	5.96	6.73	6.3
11	3.07	2.72	3.75	–
12	1.81	1.80	1.84	1.81

Table I, in comparison to the Eyring and Fitzroy methods, and the experimental values determinated.

The figures referring to graphic recorder decay curves applying to the specific rooms, are indicated from Figs. 6 to 13.

The differences existent between theoretical and experimental values of the reverberation time in the two first cases, can be due to the fact that the absorption energy coefficient of the resonators A1, A2, A3, were calculated but not measured in a reverberation chamber. In other cases the agreement is better.

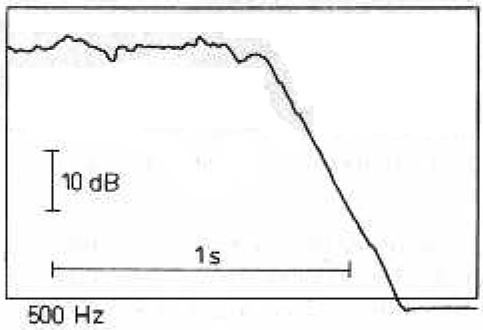

Fig. 6. Recorder graph in room 1.

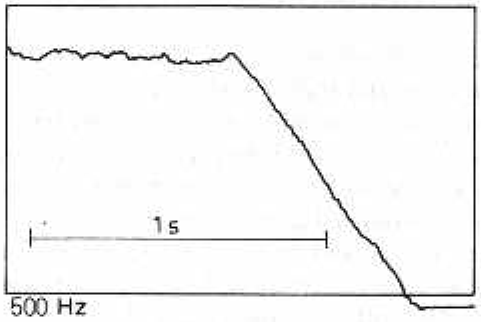

Fig. 7. Recorder graph in room 2.

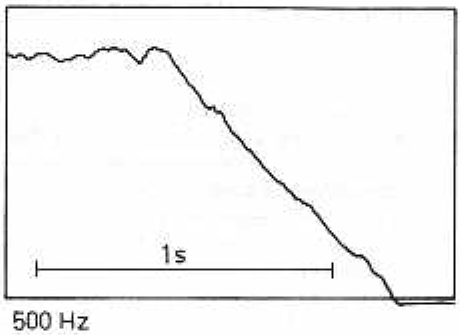

Fig. 8. Recorder graph in room 3.

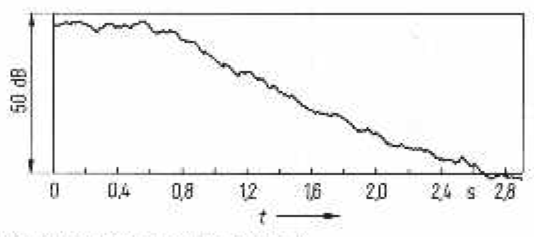

Fig. 9. Recorder graph in room 4.

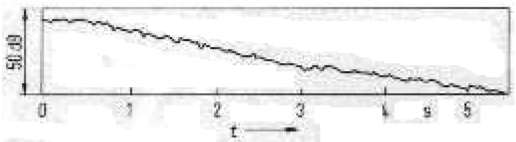

Fig. 10. Recorder graph in room 5.

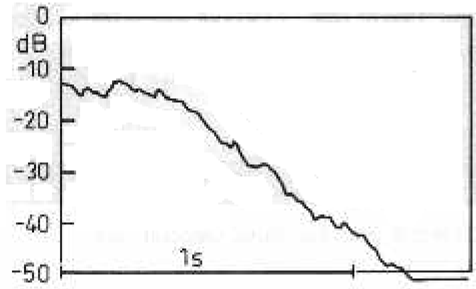

Fig. 11. Recorder graph in room 6.

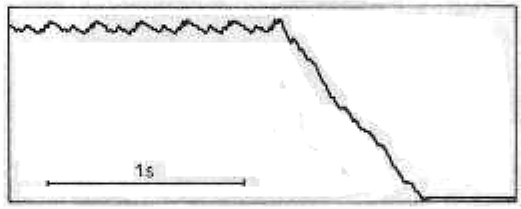

Fig. 12. Recorder graph in room 7.

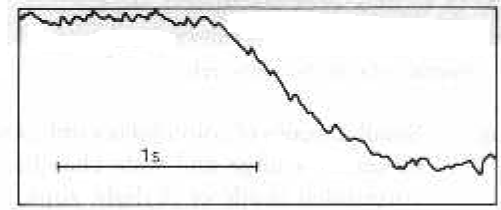

Fig. 13. Recorder graph in room 8.

5. Other examples analyzed: area and location material

In this section we will make a comparison between different cases proposed by us. With them we would may deduce important conclusions concerning our formula.

1) Let be first a cubical room of $3\,\mathrm{m} \times 3\,\mathrm{m} \times 3\,\mathrm{m}$ of dimensional ratio, placing absorbing material ($\alpha = 0.4$) on the two opposite walls. Being the remaining walls, for example: concrete, ($\alpha_i = 0.05$).

In these conditions, we have:

$$\frac{x}{S} = \frac{y}{S} = \frac{z}{S} = 0.33; \quad \bar{\alpha}_x = 0.05, \quad \bar{a}_x = 0.051;$$

$$\bar{\alpha}_y = 0.05, \quad \bar{a}_y = 0.051;$$

$$\bar{\alpha}_z = 0.40, \quad \bar{a}_z = 0.51; \quad \bar{\alpha}_{av} = 0.167.$$

Calculating the reverberation time, we find:

T_E (Eyring) $= 0.44$ s,
T_F (Fitzroy) $= 1.11$ s,
T (New formula) $= 0.73$ s.

2) Let be the same case 1) except that the absorptive material is placed on adjacent walls.

For this example we have:

$$\frac{x}{S} = \frac{y}{S} = \frac{z}{S} = 0.333; \quad \bar{\alpha}_x = 0.05, \quad \bar{a}_x = 0.051;$$

$$\bar{\alpha}_y = 0.225, \quad \bar{a}_y = 0.255;$$

$$\bar{\alpha}_z = 0.225, \quad \bar{a}_z = 0.255; \quad \bar{\alpha}_{av} = 0.167.$$

Evaluating the reverberation time, we find:

T_E (Eyring) $= 0.44$ s,
T_F (Fitzroy) $= 0.74$ s,
T (New formula) $= 0.54$ s.

Comparing this last value of T with the one obtained in the case 1), we see that the effect of the absorption is greater if given amounts of sound absorbing material are placed on adjacent walls of a cube rather than on opposite walls. This effect was observed, as explains Young [5], by C. A. Andree (1932).

3) Let be now a new room of equal chord $4V/S$ to the one described before. Its dimensions are $6\,\text{m} \times 3\,\text{m} \times 2\,\text{m}$. Supposing that we place any absorptive ($\alpha = 0.4$) material between the ceiling and floor, so such that the mean arithmetic energy coefficient of the room $\bar{\alpha}_{av}$ be the same to the before case. The remaining surfaces are of concrete ($\alpha_i = 0.05$).

We have now:

$$\frac{x}{S} = 0.500, \quad \bar{\alpha}_x = 0.284, \quad \bar{a}_x = 0.334;$$

$$\frac{y}{S} = 0.333, \quad \bar{\alpha}_y = 0.05, \quad \bar{a}_y = 0.051;$$

$$\frac{z}{S} = 0.166, \quad \bar{\alpha}_z = 0.05, \quad \bar{a}_z = 0.051;$$

$$\bar{\alpha}_{av} = 0.167.$$

The reverberation time will be:

T_E (Eyring) $= 0.316$ s,
T_F (Fitzroy) $= 0.91$ s,
T (New formula) $= 0.616$ s.

4) We consider the case 3) placing the absorbing material in side walls, being the remaining surfaces of concrete, to obtain the same $\bar{\alpha}_{av} = 0.167$.

Therefore we can write:

$$\frac{x}{S} = 0.500, \quad \bar{\alpha}_x = 0.05, \quad \bar{a}_x = 0.051;$$

$$\frac{y}{S} = 0.333, \quad \bar{\alpha}_y = 0.402, \quad \bar{a}_y = 0.515;$$

$$\frac{z}{S} = 0.166, \quad \bar{\alpha}_z = 0.05, \quad \bar{a}_z = 0.051;$$

$$\bar{\alpha}_{av} = 0.167.$$

Calculating the time of reverberation, we have:

T_E (Eyring) $= 0.316$ s,
T_F (Fitzroy) $= 1.10$ s,
T (New formula) $= 0.73$ s.

5) We will consider a room of dimensions: $30\,\text{m} \times 10\,\text{m} \times 3\,\text{m}$, treating the two bottom walls with absorbing material with $\alpha = 0.4$. The remaining walls are of concrete ($\alpha_i = 0.05$).

In this case we have:

$$\frac{x}{S} = 0.714, \quad \bar{\alpha}_x = 0.05, \quad \bar{a}_x = 0.051;$$

$$\frac{y}{S} = 0.214, \quad \bar{\alpha}_y = 0.05, \quad \bar{a}_y = 0.051;$$

$$\frac{z}{S} = 0.071, \quad \bar{\alpha}_z = 0.40, \quad \bar{a}_z = 0.51;$$

$$\bar{\alpha}_{av} = 0.075.$$

The reverberation time are:

T_E (Eyring) $= 2.22$ s,
T_F (Fitzroy) $= 3.16$ s,
T (New formula) $= 2.88$ s.

6) Let be the same case 5) removing the $60\,\text{m}^2$ of absorbing material of the bottom walls, and setting it on ceiling, assuming that the remainder surfaces are of concrete, we have:

$$\frac{x}{S} = 0.714, \quad \bar{\alpha}_x = 0.085, \quad \bar{a}_x = 0.089;$$

$$\frac{y}{S} = 0.214, \quad \bar{\alpha}_y = 0.05, \quad \bar{a}_y = 0.051;$$

$$\frac{z}{S} = 0.071, \quad \bar{\alpha}_z = 0.05, \quad \bar{a}_z = 0.051;$$

$$\bar{\alpha}_{av} = 0.075.$$

Calculating the reverberation time, we find:

T_E (Eyring) $= 2.22$ s,

T_F (Fitzroy) $= 2.35$ s,

T (New formula) $= 2.27$ s.

7) Let be a room of the following rectangular dimensions: 30 ft × 19 ft × 24 ft. This case is considered experimentally by Knudsen et al. [21]. We suppose with them a floor absorptive 560 ft sq ($\alpha = 0.74$ to 250 Hz). The remaining surfaces are reflectants ($\alpha_i = 0.024$).

In this case we have:

$$\frac{x}{S} = 0.326, \quad \bar{\alpha}_x = 0.376, \quad \bar{a}_x = 0.472;$$

$$\frac{y}{S} = 0.412, \quad \bar{\alpha}_y = 0.024, \quad \bar{a}_y = 0.024;$$

$$\frac{z}{S} = 0.261, \quad \bar{\alpha}_z = 0.024, \quad \bar{a}_z = 0.024;$$

$$\bar{\alpha}_{av} = 0.139.$$

Calculating now the reverberation time, we can write:

T_E (Eyring) $= 1.28$ s,

T_F (Fitzroy) $= 5.52$ s,

T (New formula) $= 3.42$ s.

The decay associated to the time of reverberation by our new formula, results in: 17.55 dB/s.

Comparing this theoretical result with the experimental value measured by Knudsen et al.: 17.6 dB/s, at 250 cps and microphone at 18 ft, Fig. 10, it is shown the remarkable agreement concordance between measured and predicted values.

8) Here we will describe the experiment performed by R. W. Young [5]:

We have a rectangular concrete room of volume 1350 ft^3, with sound absorbing material (area 265 ft^2) covered the ceiling and top third of side walls, at 1000 cps the effective Sabine coefficient was 0.25; but when the material was arranged in a border 1 ft wide around the ceiling area (area of the border 46 ft^2), the effective Sabine coefficient of the absorptive material was 0.93. In these conditions we have approximating:

$$\frac{x}{S} = 0.536, \quad \text{Ratio: } 11.22\,\text{ft} \times 19.56\,\text{ft} \times 6.16\,\text{ft};$$

$$\frac{y}{S} = 0.1688, \quad S\text{: } 817.1\,\text{ft}^2;$$

$$\frac{z}{S} = 0.416, \quad V\text{: } 1350\,\text{ft}^3.$$

We start assuming to be true that the effective Sabine coefficient for second case, was $a_s = 0.93$; from which we obtain the following energy absorption coefficient of material $\alpha = 0.61$:

Applying this value to the first case shown by Young, we have:

$$\bar{\alpha}_x = 0.330, \quad \bar{a}_x = 0.40; \quad \bar{\alpha}_y = 0.231, \quad \bar{a}_y = 0.268;$$

$$\bar{\alpha}_z = 0.05, \quad \bar{a}_z = 0.051; \quad \bar{\alpha}_{av} = 0.24.$$

Evaluating the eq. (6), we obtain the following decay rate absorption coefficient of the room:

$$\bar{a} = (0.40)^{0.536} \cdot (0.268)^{0.1688} \cdot (0.051)^{0.416} = 0.101.$$

Associated to this value $\bar{a}$, there is a mean energy absorption coefficient of the room given by eq. (3): $\bar{\alpha} = 0.096$ which is very different to the above indicated $\bar{\alpha}_{av} = 0.24$!

Assuming now that, $\bar{\alpha} = 0.096 = 1/s \sum s_i \alpha_i$ solving for the unknown α, we will obtain the effective value ignored of the absorptive material in its actual ubication inside of the room: $\alpha = 0.193$, and hence calculating we obtain the following effective Sabine coefficient of the absorptive material $a_s = 0.214$. We recall that Young measured 0.25 for this case.

Studying now the second case of Young, we have:

$$\bar{\alpha}_x = 0.133, \quad \bar{a}_x = 0.120; \quad \bar{\alpha}_y = 0.05, \quad \bar{a}_y = 0.051;$$

$$\bar{\alpha}_z = 0.05, \quad \bar{a}_z = 0.051; \quad \bar{\alpha}_{av} = 0.089.$$

From the eq. (6) we obtain $\bar{a} = 0.081$ that has one $\bar{\alpha}$, connected to $\bar{a}$, equal to: $\bar{\alpha} = 0.078$.

Assuming now to be true for us that $\bar{\alpha} = 0.078$, we get α of the absorbing material, from the equation: $0.078 = 1/S \sum S_i \alpha_i$.

We have obtained the following result for the material $\alpha = 0.598$ that it has associated the Sabine coefficient $a_s = 0.91$ in front of the experimental value 0.93 determinated by R. Young.

Therefore is true that the effective absorption per unit area changes with amount of material present in a room. Also, by another side, can be understandable easily for us that when the acoustical measurements are made in a reverberant chamber, on any absorbing material, the experimental values determined a_s of the material will be rights if is verified, with enough approximation, the following equality:

$$-\ln(1 - \bar{\alpha}_{av}) = \bar{a}_x^{x/S}\, \bar{a}_y^{y/S}\, \bar{a}_z^{z/S}. \tag{24}$$

This relation involves that in each different reverberant room, depending of its dimensional ratio, will must exist a maximum area of absorbing material preserving the sound diffuse condition, in the necessary amount, for which be fulfilled the eq. (24), or that let be $\bar{\alpha}_{av} \cong \bar{\alpha}$.

9) Let be the following non diffuse reverberation room: 6.80 m × 5.2 m × 5.6 m, $V = 196$ m3, $S = 203$ m2.

$$\frac{x}{S} = 0.348\,, \quad \frac{y}{S} = 0.375\,, \quad \frac{z}{S} = 0.287\,.$$

Supposing, with empty room, that the reverberation time T_0 is $T_0 = 4.02$ seconds (500 Hz), we have $\alpha_0 = 0.038$, where α_0 is the energy absorption coefficient of the walls of the room. Introducing now an absorptive material on the floor of the chamber, with an area $F = 4$ m2, we obtain the following experimental Sabine coefficient $a_s = 1.35$ for the absorbing material ($\alpha = 0.74$) occupating a surface on floor equal to

$$F/V^{2/3} = 0.12.$$

The Table III shows the values calculated, for the different coefficients, in relation to $F/V^{2/3}$.

Table III.
Variation of a_s with the absorptive occupation parameter.

File	Coefficients	Parameter: $F/V^{2/3}$					
		0.12	0.24	0.36	0.6	0.8	1
1	$\bar{\alpha}_x$	0.072	0.117	1.157	0.239	0.306	0.373
2	$\bar{\alpha}_y$	0.038	0.038	0.038	0.038	0.038	0.038
3	$\bar{\alpha}_z$	0.038	0.038	0.038	0.038	0.038	0.038
4	$\bar{\alpha}_{ey}$	0.052	0.066	0.069	0.108	0.131	0.155
5	$\bar{a}_x$	0.081	0.125	0.171	0.273	0.365	0.467
6	$\bar{a}_y$	0.038	0.038	0.038	0.038	0.038	0.038
7	$\bar{a}_z$	0.038	0.038	0.038	0.038	0.038	0.038
8	$\bar{a}$	0.049	0.057	0.063	0.074	0.082	0.090
9	$\bar{\alpha}$	0.048	0.055	0.061	0.071	0.079	0.086
10	a_s	1.35	0.653	0.557	0.466	0.425	0.396
11	$\bar{\alpha}_{ey} - \bar{\alpha}$	0.004	0.011	0.018	0.037	0.052	0.069

The values of $\bar{a}$, belonging to file 8, are calculated from eq. (6); and other hand the $\bar{\alpha}$ coefficients, shown on file 9, are deduced from file 8 values using eq. (3).

The values written in file 10 are derived from file 9 by

$$\bar{\alpha} = \frac{1}{S} \sum S_i\, \alpha_i$$

clearing up α of the absorptive material, assumed this α unknown in this step, and computing after the effective Sabine coefficient a_S from α by using the eq. (3).

In the Fig. 14 we have plotted a_S against $F/V^{2/3}$ trying to explain the dependence of a_s in correlation to the absorptive occupation parameter.

Observing the Fig. 12 we see there is a tendency to a high value of a_s for small areas F, and at values $F/V^{2/3} > 0.4$ the absorption coefficient decreases less rapidly.

Also analyzing the same figure we see that the difference between the results of a_S with $F = 12$ m^2 and with 4 m^2 is of the order of 2.

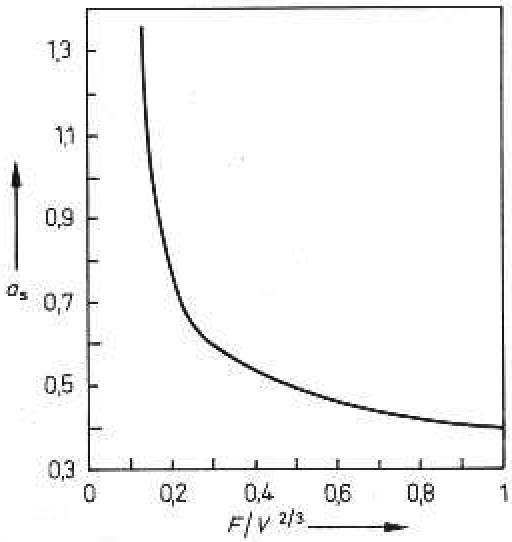

Fig. 14. a_s in function of $F/V^{2/3}$.

Both conclusions were also cited by Kosten in [22]. From Table III a potential curve fit the relation of the Sabine coefficient with the parameter $F/V^{2/3}$ is obtained:

$$a_S = 0.36 \left(\frac{F}{V^{2/3}} \right)^{-0.54}. \tag{25}$$

Starting from (25) we deduce:

$$a_S = 0.36 \left(\frac{V}{F^{1.5}} \right)^{0.36}, \tag{26}$$

This prove that a_s decreases with F and increases with V, and therefore also will be certain that to arrive at the same result of a_S in rooms of different size, it is required a larger sample in a larger room, conclusion that also it was shown experimentally by the Round Robin test [22].

Applying the eq. (26) to the example 8 analyzed by Young for $F = 265$ ft^2 and $V = 1350$ ft^3 we arrive to $a_S = 0.24$, being the value measured $a_S = 0.25$.

10) We consider now the experimental example indicated by Brüel [23].

Let be a pair of rooms of dimensions: $6.1 \times 6.2 \times 3.82 = 144$ m^3.

The first room named DB had the sound absorbing material situated at the ceiling, the rest being furniture and carpets placed on the floor.

The second room named GB the same absorbing material in equal amount was uniformly spread all over the bounding surfaces of the room.

We have assumed for the absorptive material an energy absorption coefficient $\alpha = 0.6$ for 1000 Hz, and the remaining surfaces let be $\alpha_i = 0.05$. Being $x/S = 0.446$; $y/S = 0.279$, $z/S = 0.275$ we have:

For the DB room:

$\bar{\alpha}_x = 0.310$, $\bar{a}_x = 0.370$; $\bar{\alpha}_y = 0.05$, $\bar{a}_y = 0.051$;

$\bar{\alpha}_z = 0.05$, $\bar{a}_z = 0.051$

by using (18) we establish $T = 1.1$ s, being the value measured $T = 1$ s.

For the GB room:

$\bar{\alpha}_x = 0.142$, $\bar{a}_x = 0.153$; $\bar{\alpha}_y = 0.196$, $\bar{a}_y = 0.218$;

$\bar{\alpha}_z = 0.212$, $a_z = 0.238$.

By (18) we calculate $T = 0.71$ s, being the experimental value $T = 0.68$ s.

We must remark that the theoretical results are evidently in good agreement with expectation.

11) We consider now the important experimental examples analysed by Mehta-Mulholland [24].

They worked with a rectangular room 4.5 m × 2.73 m × 2.40 m, in where measured to 1000 Hz, with empty room, a reverberation time equal to 2.2 s.

From the different cases analysed by them, I now select five cases recollecting very different conditions. Let be the following:

Case 1 d): Samples fully covering the two opposite walls 4.54 m × 2.40 m

Case 1 c): Samples fully covering the wall 4.50 m × 2.40 m

Case 1 k): Samples fully covering the floor and the two opposite walls 2.73 m × 2.40 m

Case 1 i): Samples fully covering the floor and the wall 2.73 m × 2.40 m

Case 1 e): Samples fully covering the three mutually perpendicular walls.

The sample have a absorption coefficient $\alpha = 0.86$ (1000 Hz). Following I shall make a comparison of the reverberation time among:

- Cremer (2.31 eq. ref. [7]) ≡ Millington formula for this case
- Kuttruff (2.80 eq. ref. [7])
- Eyring
- Sabine
- Fitzroy
- Arau (18 eq.)
- Experimental (obtained by Mehta-Mulholland)

for all the combinations which before I have exposed here.

In first step I write the geometrical conditions of the enclosure considered.

$$V = 29.484 \text{ m}^3;$$

$$S_1 = 12.285 \text{ (floor)}; \qquad \frac{S_1}{S} = \frac{S_2}{S} = 0.20725;$$

$$S_2 = 12.285 \text{ (ceiling)}; \qquad \frac{S_3}{S} = \frac{S_4}{S} = 0.1822;$$

$$S_3 = 10.8 \text{ (side wall)}; \qquad \frac{S_5}{S} = \frac{S_6}{S} = 0.1105;$$

$$S_4 = 10.8 \text{ (side wall)};$$

$$S_5 = 6.552 \text{ (bottom wall)};$$

$$S_6 = 6.552 \text{ (bottom wall)};$$

$$S = \sum_i S_i = 59.274;$$

$$x = S_1 + S_2 = 24.57; \qquad \frac{x}{S} = 0.4145;$$

$$y = S_3 + S_4 = 21.6; \qquad \frac{y}{S} = 0.3644;$$

$$z = S_5 + S_6 = 13.104; \qquad \frac{z}{S} = 0.2210;$$

As second step I put the absorption conditions of the different cases (Table IV):

Table IV.
Absorption and reflection coefficients of the case 11).

	1d	1c	1k	1i	1e
α_1	0.036	0.036	0.86	0.86	0.86
α_2	0.036	0.036	0.036	0.036	0.036
α_3	0.86	0.86	0.036	0.036	0.86
α_4	0.86	0.036	0.036	0.036	0.036
α_5	0.036	0.036	0.86	0.86	0.86
α_6	0.036	0.036	0.86	0.036	0.036
a_1 [1]	0.0366	0.0366	1.966	1.966	1.966
a_2	0.0366	0.0366	0.0366	0.0366	0.0366
a_3	1.966	1.966	0.0366	0.0366	1.966
a_4	1.966	0.0366	0.0366	0.0366	0.0366
a_5	0.0366	0.0366	1.966	1.966	1.966
a_6	0.0366	0.0366	1.966	0.0366	0.0366
ϱ_1 [1]	0.964	0.964	0.14	0.14	0.14
ϱ_2	0.964	0.964	0.964	0.964	0.964
ϱ_3	0.14	0.14	0.964	0.964	0.14
ϱ_4	0.14	0.964	0.964	0.964	0.964
ϱ_5	0.964	0.964	0.14	0.14	0.14
ϱ_6	0.964	0.964	0.14	0.964	0.964
$\bar{\alpha}_x = \frac{\alpha_1 + \alpha_2}{2}$	0.036	0.036	0.448	0.448	0.448
$\alpha_y = \frac{\alpha_3 + \alpha_4}{2}$	0.86	0.448	0.036	0.036	0.448
$\bar{\alpha}_z = \frac{\alpha_5 + \alpha_6}{2}$	0.036	0.036	0.86	0.448	0.488
$\bar{\alpha}_{av} = \frac{1}{S} \sum_i S_i \alpha_i$	0.3362	0.186	0.388	0.237	0.448
$\bar{\varrho} = (1 - \bar{\alpha}_{av})$	0.6638	0.814	0.612	0.763	0.552
$a_x = -\ln(1 - \bar{\alpha}_x)$	0.0366	0.0366	0.594	0.594	0.594
$\bar{a}_y = -\ln(1 - \bar{\alpha}_y)$	1.966	0.594	0.0366	0.0366	0.594
$\bar{a}_z = -\ln(1 - \bar{\alpha}_z)$	0.0366	0.0366	1.966	0.594	0.594
$\bar{a} = \bar{a}_x^{x/S}\, \bar{a}_y^{y/S}\, \bar{a}_z^{z/S}$	0.156	0.101	0.282	0.215	0.594
$\bar{\alpha}$: $[\bar{a} = -\ln(1 - \bar{\alpha})]$	0.194	0.096	0.246	0.194	0.488
$(\bar{\alpha}_{av} - \bar{\alpha})$	0.192	0.090	0.142	0.043	0
Cremer:					
α'_m (eq. 2.31 [7])	0.740	0.388	0.863	0.650	1.001
Kuttruff:					
α'' (eq. 2.80 [7])	0.442	0.223	0.551	0.291	0.653

[1] $\varrho_i = 1 - \alpha_i$; $a_i = -\ln(1 - \alpha_i)$.

Table V.
Reverberation time of the case 11).

Reverberation time	1d	1c	1k	1i	1e
Experimental [1]	0.52	0.71	0.29	0.40	0.17
T_A: Arau	0.51	0.79	0.28	0.37	0.14
T_S: Sabine	0.24	0.43	0.21	0.34	0.18
T_E: Eyring	0.20	0.39	0.16	0.23	0.14
T_K: Kuttruff	0.18	0.36	0.15	0.28	0.12
T_M: Millington	0.11	0.21	0.09	0.12	0.08
T_F: Fitzroy [2]	1.31	1.36	0.81	0.84	0.14

[1] Measured by Mehta-Mulholland
[2] Calculated by Mehta-Mulholland

Calculating the reverberation time and comparing the several theories in front of the experimental values, for the combinations described, we have from better agreement to worse the following results (Table V).

Let see us again the good agreement of the results obtained from our new formula (18) with the experimental values measured by Mehta-Mulholland.

5. Conclusions

In this paper we have developed a new reverberation formulae, illustrated with many cases and examples, that permit us to obtain important conclusions above the effects of area and location of absorptive material in a room. We have demonstrated the experimental work of Young and Knudsen and others, as also the good agreement of our experimental results with theoretical work, that confirm the validity of our method treating the non-uniform distribution of absorption in rooms.

We deduce that the empirical Fitzroy's formula only should have physical sense in those especial cases of rooms in where the reverberation periods T_x, T_y and T_z are near to 0.5 s, that in conjunction with the percentages of area values, enable us to keep also T nearly to 0.5 s. Under this hypothesis taking logarithms in eq. (18), or (16), and approximating $(-\ln T_i)$ by T_i on each term, we arrive to the Fitzroy's formula:

$$T = \frac{x}{S} T_x + \frac{y}{S} T_y + \frac{z}{S} T_z \,. \tag{2}$$

This last formula has been the live-motive inspiring our simple investigation; and with it we have tried to explain some aspects almost unknown of the sound behavior on rooms.

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