A comparative analysis of acoustic energy models for churches

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(Received 3 February 2009; revised 17 July 2009; accepted 24 July 2009)

Different models to improve prediction of energy-based acoustic parameters in churches have been proposed by different researchers [E. Cirillo and F. Martellotta, J. Acoust. Soc. Am. **118**, 232–248 (2005); T. Zamarreño *et al.*, J. Acoust. Soc. Am. **121**, 234–250 (2006)]. They all suggested variations to the "revised" theory proposed by Barron and Lee [J. Acoust. Soc. Am. **84**, 618–628 (1988)], starting from experimental observations. The present paper compares these models and attempts to generalize their use taking advantage of the measurements carried out in 24 Italian churches differing in style, typology, and location. The whole sample of churches was divided into two groups. The first was used to fine-tune existing models, with particular reference to the "μ model," which was originally tested only on Mudejar-Gothic churches. Correlations between model parameters and major typological and architectural factors were found, leading to a classification that greatly simplifies parameter choice. Finally, the reliability of each model was verified on the rest of the sample, showing that acoustic parameters can be predicted with reasonable accuracy provided that one of the specifically modified theories is used. The results show that the model requiring more input parameters performs slightly better than the other which, conversely, is simpler to apply.
 [DOI: 10.1121/1.3205398]

PACS number(s): 43.55.Br, 43.55.Gx [NX] Pages: 1838–1849

I. INTRODUCTION

Theoretical models to predict acoustical parameters have been attracting renewed interest in recent years, with particular reference to churches. Computer simulations may provide detailed and reliable results, but they require a detailed three-dimensional model of the room under investigation, which may be a quite difficult task, especially for buildings such as churches. Conversely, theoretical models interpreting how sound propagates in such complex spaces provide simple prediction formulas to calculate reference values with little computational effort, aiding the general understanding of room acoustics in the process.

An energy model generally defines a law that describes the way in which sound energy propagates. These models can be divided into two categories: the theoretical ones, based on the interpretation of the sound field and on the definition of mathematical laws capable of describing its variations, and the empirical or semi-empirical ones, derived from correlations of data. The theoretical approach is generally more sophisticated, even though more complex, while the empirical approach gives relations that, even without a substantial improvement of the understanding of the physical aspects, may be as accurate as the others. A key issue that these models should satisfy is their reasonable ease of use. In fact, the success of the reverberation time among the acoustic descriptors of a space relies not only on its correlation with perceived subjective quality, but also on its steadiness throughout the space, and, above all, on its predictability with simple formulas,[1,2] which cover most of the cases, even churches. However, a number of other acoustic parameters have been defined in order to better match subjective perception of the sound field, even though the prediction of such acoustic parameters depends on many factors, such as the relative position of sources and receivers and any mathematical formulation becomes more complex.

According to the classical theory of diffuse sound propagation in rooms with uniformly distributed absorption, the sound pressure level is the sum of a direct component and a diffuse one. Barron and Lee[3] proposed a "revised theory" assuming that the reflected sound cannot arrive earlier than direct sound. This theory was proposed for concert halls or other proportionate spaces,[4] providing considerably better predictions of the actual behavior. A modification of this theory was proposed by Vörlander[5] for reverberant chambers.

Unfortunately, several studies[6–10] show that churches and other places of worship can hardly be included among the proportionate spaces. Measurements of both strength and clarity carried out in Mudejar-Gothic churches,[6–8] in mosques,[9] and in Italian churches[10] show that reflected sound level is below that predicted by previous theories. Possible explanations of this difference may be found in the "disproportionate" nature of these kinds of buildings, in the non-uniform distribution of sound absorbing materials, and in their architectural elements such as side aisles, chapels, vaults, and domes. These elements scatter or hinder the sound, especially affecting the early reflections where the energy value is greater. A simple empirical approach is to obtain prediction equations based on simple regression formulas.[11] However, the above-mentioned works[6–10] show that, despite some fluctuations, the acoustic energy parameters are well related to source-receiver distance. So, researchers attempted to modify the revised theory in order to take into account, in a more or less empirical way, the physical phenomena, that cause the early reflections to decrease. Those models were validated on a specified group of Spanish

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churches[6,12] and on a wider sample of Italian churches,[13,14] leading to improved prediction accuracy. A brief overview of such models is reported in Sec. II.

Starting from the results of a preliminary study where the more recent model defined for Spanish churches[12] was used on a large sample of Italian churches,[15] the present work is aimed at investigating the possibility of generalizing this model to different churches by means of a typological classification. In addition, differences between models and, where possible, their similarities were investigated. Finally, the prediction accuracy of different models was analyzed through a comparison between measured and predicted values of sound strength, clarity, and center time.

II. OVERVIEW OF ENERGY MODELS

A. The classical theory and the "revised" theory

According to the classical theory of sound propagation in enclosed rooms,[1] if the absorption is uniformly distributed and if the sound field is diffuse, the relative sound pressure level (also known as strength, G) at a distance r from the source, assuming as a reference the level of the direct sound at a distance of 10 m from the source, and expressing total acoustic absorbing area as a function of the reverberation time (T) and of the room volume (V), through Sabine's equation is

$$G(r) = 10 \log(100/r^2 + 31\,200T/V) \quad [\text{dB}]. \tag{1}$$

However, according to Eq. (1), when the direct sound becomes negligible, G only depends on T and V, and should be the same throughout the space. Other energy-based acoustic parameters, such as clarity (C_{80}) and center time (T_S), may also be calculated by taking into account that the instantaneous reverberant energy follows a decay function that decreases exponentially (with a time constant $T/13.8$) and, after integration, must yield the diffuse-field contribution in Eq. (1) as follows:

$$g(t) = (13.8 \times 31\,200/V)e^{-13.8t/T} \quad [\text{s}^{-1}]. \tag{2}$$

However, as observed for G, the resulting values of the parameters are substantially independent of the distance.

The classical formulation shows limited effectiveness in predicting the acoustic behavior in large rooms as measured acoustic parameters show much larger variations as a function of the distance from the source. Barron and Lee[3] found that the sound level decay was linear soon after the direct sound in the majority of the halls and the reflected sound level decreased with increasing source-receiver distance. So, they proposed a model based on the following four assumptions. (i) The direct sound is followed by linear level decay at a rate corresponding to the reverberation time. (ii) The instantaneous level of the late decaying sound is uniform throughout the space. (iii) The time $t=0$ corresponds to the time the signal is emitted from the source; therefore the direct sound reaches a point at a distance r from the source after a time $t_D=r/c$. In this way the integrated energy decreases when the source-receiver distance increases, while the early/late reflected energy ratio remains constant. (iv) The integrated value for the reflected sound level is assumed to be, at $r=0$, equal to the value predicted by the classical theory [Eq. (1)].

According to the revised theory, the integrated reflected energy from time $\tau=t-t_D$ (i.e., assuming the time origin at $t=t_D$) to infinity, at a point at a distance r from the source is given by

$$i(\tau,r) = (31\,200T/V)e^{-0.04r/T}e^{-13.8\tau/T}. \tag{3}$$

In order to ease the calculation of the clarity index, the sound energy is divided into three components: the direct sound (d), the early reflected sound (from 0 to 80 ms, E_0^{80}), and the late reflected sound (from 80 ms to infinity, E_{80}^{∞}). From Eq. (3), the corresponding energies become

$$d(r) = 100/r^2, \tag{4}$$

$$E_0^{80}(r) = (31\,200T/V)e^{-0.04r/T}(1 - e^{-1.11/T}), \tag{5}$$

$$E_{80}^{\infty}(r) = (31\,200T/V)e^{-0.04r/T}e^{-1.11/T}. \tag{6}$$

So G, C_{80}, and T_S may be calculated according to Eqs. (10)–(12) given in Ref. 13.

B. The "μ model" for Mudejar-Gothic churches

The lack of accuracy of the revised theory when applied to churches was first pointed out by Sendra *et al.*,[6] who proposed an empirical correction of the theory, known as the β model. This model replaced the coefficient 0.04 (resulting from $13.8/c$) appearing in Eq. (3) with a coefficient β. The application of the model to some churches led to the empirical estimation of β values providing the best accuracy in predicting G. The resulting values varied between 0.06 and 0.12, suggesting a loss of energy that the authors attributed to the geometrical complexity of the churches.

However, as observed by Zamarreño *et al.*,[12] this model proved to be quite ineffective in predicting other monaural parameters, mostly because the β coefficient equally affects both early and late reflected energies. Taking advantage of measurements carried out in ten Mudejar-Gothic churches the same authors proposed an alternative approach capable of providing improved prediction accuracy. According to the new formulation the empirical coefficient, named μ, only affects the early part (from 0 to 80 ms) of the reflected sound. The μ coefficient is then derived from regression analysis in order to minimize the differences between measured and estimated values of a given acoustical parameter. This assumption is aimed at improving the agreement with measured values, even though it determines a discontinuity in the reflected energy function [Fig. 1(a)]. The authors justify this discrepancy by the discrete nature of the early reflections so that they consider it unnecessary to assume a continuous function.

According to the new approach only Eq. (5) needs to be rewritten as follows:

$$E_{\mu 0}^{80}(r) = (31\,200T/V)e^{-\mu r/T}(1 - e^{-1.11/T}), \tag{7}$$

so that G and C_{80} may be calculated by simply replacing $E_{\mu 0}^{80}$ with E_0^{80}, while center time needs to be rewritten to account

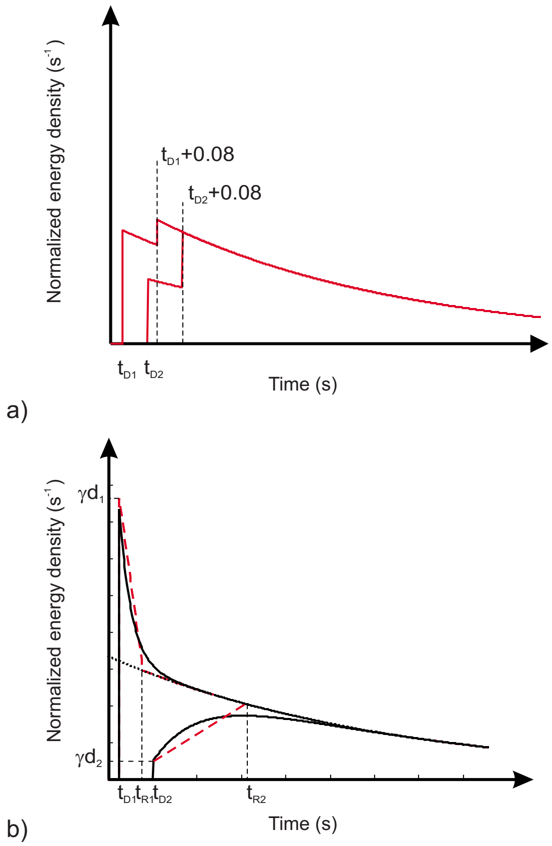

FIG. 1. (Color online) (a) Shape of the energy decay curve obtained with μ model observed at different distances from the source. The early reflected energy (from t_D to t_D+0.08 s) is given by an exponential energy decay having the same time constant of the late part but estimated at a later time (multiplied by $\mu/0.04$). (b) Shapes of the energy decay curve obtained with the linear model (---) and with the multi-rate exponential model (—) observed at different distances from the source. Dotted curve represents classical exponential decay.

for the displacement of the center of mass of the early reflected energy:

$$T_s(r) = \{[T - 1.11/(e^{1.11} - 1)]E_{\mu 0}^{80} + (1.11 + T)E_{80}^{\infty}\}/[13.8(d + E_{\mu 0}^{80} + E_{80}^{\infty})]. \quad (8)$$

Assuming that prediction error on G values predicted by the revised theory is relatively low, the authors proposed calculating μ values by minimizing the prediction error on C_{80}. In fact, this parameter shows the greatest variations inside a room, partly due to the arbitrary definition of the 80 ms limit for energy integration. The values calculated for Mudejar-Gothic churches show an average value of about 0.13, with a standard deviation of 0.02, so the authors assumed the value of 0.13 as specific for this group of churches. The use of the average μ gives reliable predictions of both G and T_S.

C. The "modified" theory

The analysis of the acoustic results measured in a sample of Italian churches[10,13] showed that the basic hypothesis of the revised theory, that is, the uniformity of the reverberant sound field throughout the space, was generally satisfied. However, the time at which the decay began to be linear

TABLE I. Values of the mean scattering coefficient according to the chancel typology.

Chancel typology	s
Raised, or bounded by very close hard, flat surfaces	0.20
Slightly raised, bounded by relatively flat reflecting walls with few obstacles between source and walls	0.40
Very slightly raised, bounded by scattering (decorated) walls with some obstacles between source and walls	0.60
Not raised, bounded by distant reflecting walls and full of scattering furniture	0.80

was later, the farther the measurement position was from the source. Furthermore, at points near the source, the early reflections brought more energy than the ideal classical reverberant field while conversely, when the distance from the source grew, the early reflections became weaker.

In order to fit with these observations two modifications were introduced. The first one was to assume the reverberant sound field to be uniform, as it is in Barron and Lee's theory,[3] but that linear level decay starts with a certain delay (t_R) after the arrival of the direct sound. The measurements showed that this delay was proportional to the source-receiver distance; therefore, in general, it could be written as $t_R = kt_D$. The k coefficient depended on the room characteristics and it was demonstrated that it could be expressed as a function of the architectural features of each church, assuming integer values from 1 to 3 growing with church complexity (Table III in Ref. 13).

The second modification was to schematize the early reflected sound arriving between the direct sound and the reverberant sound field as a continuous linear function varying from an initial value (at time t_D), proportional through a factor γ to the energy of the direct sound, and a final value (at time $t_D + t_R$), equal to the energy of the reverberant field at the same time [Fig. 1(b)]. The factor γ depended on the mean absorption coefficient (α), the mean scattering coefficient of the surfaces close to the source (s), and on the mean free path. The estimation of the mean scattering coefficient was simplified by assigning values varying from 0.2 to 0.8 as a function of the mean characteristics of the area surrounding the sound source (Table I).

However, as observed by Zamarreño *et al.*,[12] the assumption of a linearly decreasing energy density made the mathematical formulation of the model a bit complex, with particular reference to the subdivision of the reflected energy into the early and late contributions. Taking advantage of a more detailed analysis of the fine structure of the early reflections, Martellotta[14] proposed expressing the reflected energy function in the form of a double-rate decay as a linear combination of two exponential decay functions. This gives a more elegant mathematical formulation and a considerable simplification of the calculations without any loss in accuracy. According to this refined model Eq. (2) may be rewritten as follows:

$$g'(t) = A_1 e^{-13.8t/T_1} + A_2 e^{-13.8t/T_2}, \quad (9)$$

where $T_1 = T$ and $A_1(r) = (13.8 \times 31\ 200/V)e^{-0.04r/T}$, so that the first exponential decay coincides with Barron and Lee's,[3] while T_2 and A_2 need to be adapted in order to fit the modified linear function [Fig. 1(b)]. As explained in Ref. 14, a convenient choice is to assume that $T_2=6.9t_R$ (so that the center of gravity of the second exponential falls in the middle of the t_R interval), and $A_2(r)=\gamma d-(13.8\times 31\,200/V)e^{-0.04r/T}$, so that the initial value of the function g' is still γd. According to the proposed modifications the early and late energies (respectively, $E_0'^{80}$ and $E_{80}'^{\infty}$) can be rewritten as reported in Eqs. (13) and (14) in Ref. 14. In this way G and C_{80} may be calculated by simply replacing E with E' in Eqs. (7) and (8), while T_S needs to be calculated using Eq. (15) in Ref. 14 in order to account for the different positions of the centers of gravity of the two exponential functions. The new shape of the energy curve causes a slight decrease in the early energy at points close to the source, and an increase at farther points [Fig. 1(b)]. Consequently, it overestimates clarity and slightly underestimates center time in comparison with the "old" linear model.

III. THE ACOUSTIC SURVEY

A. Measurement technique

The measurements were carried out using an omnidirectional sound source made up of 12 120 mm loudspeakers (with a flat response from 100 Hz up to 16 kHz) mounted on a dodecahedron, together with an additional sub-woofer to cover the frequencies from 40 to 100 Hz. A calibrated measurement chain consisting of a GRAS 40-AR omnidirectional microphone together with a 01 dB Symphonie system was used to measure the sound pressure levels. A MLS signal was used to get the calibrated impulse responses to obtain the strength values. The other acoustic parameters were obtained by using high-quality impulse responses collected with a Soundfield Mk-V microphone, an Echo Audio Layla 24 sound card, and a constant envelope equalized sine sweep[16] to excite the room. Both the microphones had a flat response within the frequency bands considered in the present paper.

At least two source positions were used in each church. The source was placed 1.5 m above the floor. Ten receiver positions were used on average. In very large but symmetrical churches the receivers were only placed in one-half of the floor; otherwise they were spread to cover the whole floor area uniformly (Fig. 2). The microphone was placed 1.2 m from the floor surface. All the measurements and the calculations of the indices were made in unoccupied conditions, according to the ISO-3382 standard.[17] In particular, for the measurement of the sound strength (G) the sound power of the source was calibrated in a reverberation chamber, employing the same measurement chain and the same settings used during the on site survey.

B. The churches surveyed

Twenty-four churches located in Italy were considered in the present survey. The churches were chosen in order to include different typologies of buildings for age, style, dimensions, volume, and interior finishes. The whole sample of churches was divided into two sub-sets. The first one, including the same churches used in Ref. 13, was employed

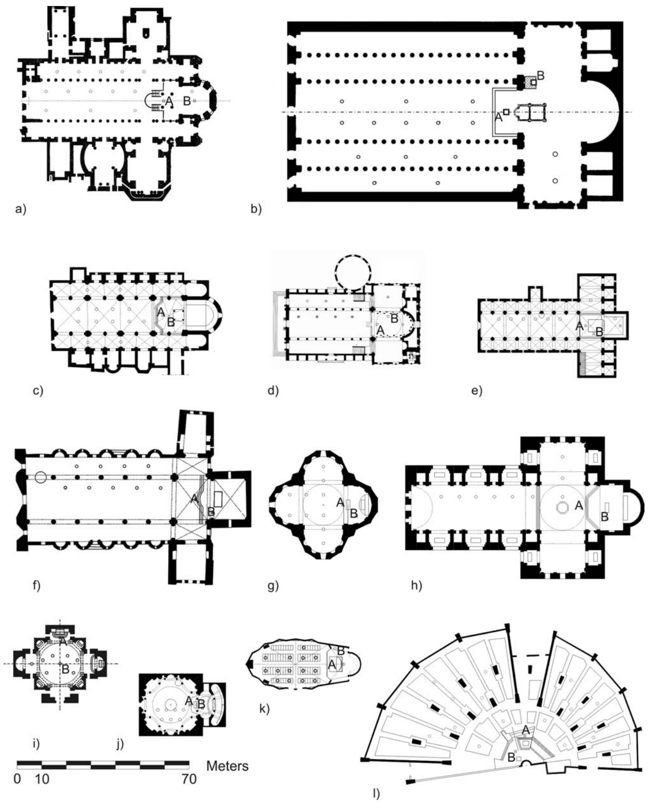

FIG. 2. Plans of the second set of 12 churches surveyed. (a) Basilica of Santa Maria Maggiore in Rome, (b) Basilica of Saint Paul outside-the-walls in Rome, (c) Sant'Ambrogio in Milan, (d) Bari Cathedral, (e) Abbey of Chiaravalle della Colomba in Alseno, (f) Orvieto Cathedral, (g) Santa Maria della Consolazione in Todi, (h) Sant'Andrea in Mantua, (i) Sant'Agnese in Agone in Rome, (j) San Lorenzo in Turin, (k) Santa Maria del Carmelo in Bari, and (l) Padre Pio Pilgrimage church in San Giovanni Rotondo. (Same scale for all the churches.)

to find a general rule to assign values for the μ model, while the second sub-set was used to compare the performances of all the models when parameter values were assigned.

For the sake of brevity the plans and descriptions of the first 12 churches are not included in this paper, as they can be found in Ref. 13, but Table II gives a summary of the most important architectural data.

The churches belonging to the second group were chosen adopting the same criteria used for the first one. In this way, even though the churches were clearly different, they could be classified according to the same geometrical and architectural characters (Table III). A short description of these churches is provided below; more detailed information may be found in Ref. 18.

The Basilica of Santa Maria Maggiore in Rome [Fig. 2(a)] is an Early-Christian church built at the beginning of the fifth century. It is built according to the typical basilica plan with side aisles even though several side chapels have been added over the centuries. It is covered with a coffered wooden ceiling with deep carvings, while the aisles are vaulted.

The Basilica of Saint Paul outside-the-walls [Fig. 2(b)] was one of the earliest churches built in Rome and, after a fire in the 1830s, it was faithfully rebuilt. The interior is a typical basilica plan 100 m long subdivided in a central nave

TABLE II. Basic details of the first set of churches surveyed.

Church	ID	Period	Style	Volume (m^3)	Total area (m^2)	Length (m)	$T_{1\,kHz}$ (s)
St. Sabina Basilica, Rome	SSA	432	Early-Christian	17 500	6 000	52	4.1
St. Apollinare in Classe, Ravenna	SAP	549	Byzantine	22 500	7 200	57	3.6
Modena Cathedral (Duomo)	MOD	1099	Romanesque	20 000	8 000	62	5.0
St. Nicholas Basilica, Bari	SNI	1197	Romanesque	32 000	10 500	59	4.4
Lucera Cathedral	LUC	1301	Gothic	33 100	10 500	64	5.3
St. Petronius Basilica, Bologna	SPB	1390	Gothic	160 000	42 000	130	9.8
Basilica Laurentiana, Florence	BLA	1419	Renaissance	39 000	18 000	82	7.9
The Holy Name of Jesus, Rome	JES	1568	Renaissance	39 000	13 000	68	5.1
St. Luca and Martina, Rome	SLM	1664	Baroque	8 700	3 500	35	3.1
St. Martin Basilica, Martina Franca	SMA	1763	Baroque	16 400	6 500	45	6.9
Concattedrale, Taranto	CCT	1970	Modern	9 000	6 200	50	4.2
S. Maria Assunta Church, Riola	RIO	1978	Modern	5 500	3 700	35	6.1

and four aisles by granite columns. The nave and the aisles are covered with a coffered ceiling made of wood.

The Basilica of Sant'Ambrogio in Milan [Fig. 2(c)] is an example of Lombard-Romanesque architecture. The interior is a conventional basilica plan without transept and with apses. The central nave is made of four square spans emphasized by cluster columns and ribbed cross vaults. The aisles are cross vaulted and topped with galleries.

Bari Cathedral [Fig. 2(d)] is a typical Apulian-Romanesque church, characterized by a basilica plan, with a main nave and two side aisles, a transept that is slightly larger than the total church width, and three semi-circular apses. The nave is flanked by marble columns alternated with pillars supporting rounded arcades, and covered by a roof with trusses.

The Abbey of Chiaravalle della Colomba in Alseno [Fig. 2(e)] is a typical Cistercian church following rigorous geometrical proportions. The walls, the pillars, and the ribs are made of facing bricks, while the vaults are finished in plaster. The transept is large, having the same width of the nave. The walls are free of decoration apart from some paintings.

The Cathedral of Orvieto [Fig. 2(f)] is a clear example of an Italian Gothic church. The interior is organized on a basilica plan, divided into three naves by ten columns and two pillars with richly decorated capitals supporting rounded arcades. The nave and aisles are covered by a wooden roof with trusses, while the transept and the choir are covered with ribbed cross vaults painted with frescoes.

The church of Santa Maria della Consolazione in Todi [Fig. 2(g)] reflects the ideal principles theorized by Renaissance architects. The Greek-cross plan derives from the combination of a square and four semi-circular apses topped by a dome on a high tambour. The decorations are shallow and made of stone.

The church of Sant'Andrea in Mantua [Fig. 2(h)] was built by Alberti in Renaissance style. The interior is a Latin cross, with a single nave flanked by four deep chapels on each side, alternating with smaller chapels. The nave is topped by an impressive barrel vault painted to simulate a coffered effect. The walls are finished in plaster and painted with rich decorative patterns.

The church of Sant'Agnese in Agone in Rome [Fig. 2(i)] is a typical baroque church, based on a central plan with elongated transversal braces and niches on the diagonals. Eight Corinthian columns in marble support the dome, emphasizing the octagonal shape of the central volume. The lower part of the church is finished in marble and stuccoes, while the upper is painted with frescoes and gilded decorations.

The church of San Lorenzo in Turin [Fig. 2(j)] was built

TABLE III. Basic details of the second set of churches surveyed.

Church	ID	Period	Style	Volume (m^3)	Total area (m^2)	Length (m)	$T_{1\,kHz}$ (s)
S. Maria Maggiore, Rome	SMM	410	Early-Christian	39 000	12 000	80	4.1
St. Paul outside-the-walls, Rome	SPX	383	Early-Christian	130 000	33 650	130	7.5
Sant'Ambrogio, Milan	SAB	1099	Romanesque	23 000	10 200	67	5.7
Cathedral of San Sabino, Bari	BAC	1100	Romanesque	30 150	9 500	59	4.8
Abbey of Chiaravalle, Alseno	ACC	1136	Gothic	13 500	7 500	59	5.6
Orvieto Cathedral	ORV	1308	Gothic	68 000	15 000	90	6.9
S. Maria della Consolazione, Todi	TOD	1508	Renaissance	19 000	4 400	39	7.9
Sant'Andrea, Mantua	SAD	1472	Renaissance	78 000	19 500	100	8.8
Sant'Agnese in Agone, Rome	SAA	1672	Baroque	14 500	5 300	28	5.0
San Lorenzo, Turin	SLO	1680	Baroque	12 000	4 500	34	4.0
Church of Carmelo, Bari	SMC	1960	Modern	9 700	3 000	46	4.2
Padre Pio Church, San Giovanni Rotondo	SGR	2004	Modern	51 000	15 600	56	5.5

by Guarini according to a central plan with a complex structure obtained by combining eight convex sides. On each side there are Corinthian columns supporting convex arches. The central space is characterized by the unique design of both the tambour and the dome with lantern.

The church of Santa Maria del Carmelo in Bari [Fig. 2(k)] is a contemporary church characterized by an almost elliptical space, even though the side walls are fragmented in order to prevent focusing effects. The roof is covered with perforated panels mostly absorbing low frequencies.

The Padre Pio Pilgrimage church in San Giovanni Rotondo [Fig. 2(l)] was designed by Piano. The church is organized according to a spiral movement focused on the chancel area, from which a series of stone arches of decreasing height radiates to hold the curved roof. The latter is made of wood and is supported by a sub-structure made of steel and wood. The interior part of the ceiling is finished with thin gypsum panels.

TABLE IV. Values of the μ parameter in the first set of churches surveyed, and corresponding rms errors in the prediction of strength, clarity, and center time.

Church	μ	rms error G (dB)	rms error C_{80} (dB)	rms error T_S (ms)
SSA	0.17	0.65	1.14	21.0
SAP	0.13	0.51	0.83	14.9
MOD	0.28	0.81	1.32	37.8
SNI	0.23	0.58	1.12	23.9
LUC	0.29	0.79	1.50	53.3
SPB	0.42	0.96	1.49	65.9
BLA	0.22	0.59	1.14	62.4
JES	0.34	1.32	1.52	50.5
SLM	0.23	0.71	1.12	19.5
SMA	0.33	2.22	1.39	113.7
CCT	0.17	1.94	0.95	48.2
RIO	0.16	0.47	0.99	25.8
Mean		0.96	1.20	44.7

IV. A TYPOLOGICAL CLASSIFICATION FOR THE MODELS

A. Generalization of μ model

At the end of their study, Zamarreño *et al.*[12] expressed the hope to extend the model "to other types of closed rooms (…) and to establish μ values for different typologies." A preliminary investigation[15] showed that with changing church typology, the resulting μ values vary over a wider range than observed in Ref. 12, suggesting that a proper choice of the parameter might allow a generalization of the model. In order to check this possibility the μ model was applied to the first 12 churches. The analysis took into account C_{80} values, at a frequency of 1 kHz as a function of source-receiver distance. For each church the μ value, which minimizes the rms error between measured and predicted values, was found by numerical iteration. Predicted values were determined using room volume and measured reverberation time at the frequency of 1 kHz.

The values found in each of the churches surveyed are reported in Table IV. They vary in the interval from 0.13 to 0.42, respectively, corresponding to the churches of Sant'Apollinare in Classe (Ravenna) and St. Petronius Basilica (Bologna). The interval shows a much greater variability than in the study of Mudejar-Gothic churches.

The analysis of rms errors for sound strength, clarity, and center time shows that using μ values specifically derived for each church leads to reasonably accurate estimates of the acoustic parameters. According to the theory, μ can be considered as an attenuation coefficient that reduces the early reflected energy by increasing the source-receiver distance fictitiously. In the analyzed churches, μ is up to ten times greater than the coefficient originally used in the revised model, equal to 0.04. This observation suggests that in churches the early reflected energy is considerably lower than that predicted by the revised theory, consequently explaining the bad performance of the latter approach.

The observation that μ is quite stable in the homogeneous sample of Spanish churches, which was used to define the model, suggests that similar values of the parameter should be expected in churches with similar characteristics. A careful analysis of the correlations between μ values and architectural aspects hopefully might lead to a classification similar to that proposed for the modified theory, allowing generalization. A typological classification of religious buildings, although necessary for acoustical purposes, represents a difficult task. The μ model depends on a single parameter (excluding V and T), and this parameter should take into account both geometry and material distribution; therefore, it is somewhat equivalent to a combination of both the parameters considered in the modified approach. Those parameters have a well-defined physical meaning, related to architectural features, so the possible relationship between them and μ was investigated. A weak correlation appeared but the statistical significance was quite low ($R^2=0.421$, with a residual probability $p=0.022$), mostly because of the discrete nature of the k parameter.

A further attempt was made to relate μ with the stylistic characteristics of the buildings. The sample was divided according to the architectural styles and the average value of μ was calculated for each one. Taking into account the strong typological connotation that characterizes some architectural styles, and the specific use of materials or ornamentations, it is possible to interpret the influence of both these aspects. In particular, lower μ values were observed in compact churches, growing when the geometrical articulation increases.[15] Early-Christian churches often have a basilica plan with few decorations and many smooth wall surfaces, their spaces are substantially open because of the limited depth of the side aisles and the slenderness of the columns. Consequently, their mean μ was 0.15. Romanesque churches are finished using nearly the same materials used in early-Christian period, but the plan is often more complex, generally with deeper aisles, so the average μ value was 0.25. Gothic churches have much larger volumes, especially because of their height and their greater spatial complexity (mostly due to lateral chapels and thick pillars). These elements determine a considerable lack of early reflections with

TABLE V. Classification of μ values according to typological style, with corresponding examples, and mean values.

Typological style	Examples	Measured range	Interval	Mean μ
Auditorium-like church or basilica with narrow aisles	SSA, SAP, CCT, and RIO	0.13–0.17	0.12–0.20	0.16
Typical basilica plan with deep aisles or thicker columns	MOD, SNI, and BLA	0.22–0.28	0.20–0.28	0.24
Basilica plan with deep aisles with large transept or chapels between source and receiver	LUC, JES, and SMA	0.29–0.34	0.28–0.36	0.32
Basilica plan with deep aisles with transept and chapels and very high ceiling/vaults	SPB	0.42	0.36–0.44	0.40

high μ values (average 0.35) compared to the other styles. In Renaissance churches, the basilica plan and an average amount of decoration led to an average μ value of 0.28. Baroque buildings are characterized by a greater spatial complexity, with side chapels, domes, and often a Greek-cross plan. These elements determine a lack of uniformity in the sound field, so their average μ value was again 0.28. Modern churches have large reflecting surfaces, due to the use of rigid materials such as concrete, which, combined with their simpler plans, determined an average μ value of 0.16.

In conclusion, studying how μ varies as a function of stylistic aspects confirms that different plan typologies, together with materials and architectural characteristics, can significantly contribute to determining the most suitable μ value for a given church. Table V shows a classification of the analyzed churches according to their architectural typologies, with the corresponding intervals of variation for calculated μ values. According to the proposed classification it appeared reasonable to subdivide the full range of variation into equally spaced intervals, and to assign the corresponding mean values to each category. Among the analyzed churches, only St. Luca and Martina in Rome did not fit the classification in Table V, probably because having a central plan and a high dome may be hardly compared with the other churches. The typological difference between the latter church and the others suggests that some adjustments in the classification are required in order to obtain a full generalization. Nonetheless, the observed results demonstrate that the classification reported in Table V gives a predictive character to the μ model, so that when the spatial articulation is known, the corresponding μ may be chosen and the energetic parameters may be calculated at each point.

It is interesting to observe that the subdivision into equally spaced intervals led to a linear growth of the mean μ value assigned to each interval, suggesting that, starting from the reference value of 0.16, any addition of given architectural elements might correspond to an increase of 0.08 in the μ value. Taking into account that an increase in μ corresponds to a subtraction of early energy, and that the addition of architectural elements between source and receiver leads to a weakening of the early reflections (both because of scattering or masking), this incremental approach appears also physically consistent. For example, the presence of either the transept or chapels in a basilica plan increases μ by 0.08, while the simultaneous presence of both increases μ by 0.16. This "incremental" approach is particularly interesting in order to deal with peculiar combinations of architectural elements, not included in Table V.

The proposed typological classification was first verified on the same group of churches in order to determine whether the errors resulting from using group values for μ instead of church-specific values were acceptable. With clarity being the parameter on which μ estimation was based, its values were calculated using the new typological values given in Table V. The resulting prediction error was slightly increased in comparison with the corresponding errors obtained using church-specific coefficients. However, the variations were generally negligible with a maximum of 0.2 dB observed in just one case and an average variation of 0.03 dB, confirming that the proposed classification can be conveniently used to generalize the μ model.

B. Toward a unified typological classification for different models

The typological classification of μ values proved to be quite effective in assigning a suitable value to a church starting from the analysis of its architectural features. Furthermore, the incremental approach appeared to be very promising in its ability to ease the difficult task of deciding which typology is best suited for a given church, in particular, when dealing with churches having complex or unusual shapes. Given this premise, a similar approach can also be proposed for the modified model in order to limit the number of arbitrary decisions and provide a unified typological classification for both models.

As already mentioned the modified model takes into account two parameters: s, which is related to the characteristics of the "sending end" of the church, and k, which is strictly related to the church typology. Consequently, a unified treatment based on typological classification should take into account μ and k as they both refer to typological features.

The classification reported in Ref. 13 defined three classes for k, which could assume integer values varying from 1 to 3. However, experimental values of this parameter varied between 1.24 (in the church of Santa Sabina in Rome), and 3.54 (in Lucera Cathedral), with typological values often differing significantly from experimental values, suggesting that despite the greater simplicity resulting from using only integer values, this approximation could lead to less accurate classification.

Taking into account the three churches belonging to the first class of the μ classification, the resulting average of experimental k values is 1.39. A straightforward extension of the μ classification to k values could then be to assign four classes of values from 1.4 to 3.5 with increments of 0.7. However, in order to take advantage of the incremental ap-

TABLE VI. Incremental values of k and μ according to the architectural characteristics.

Typical features	$k=t_R/t_D$	μ
Reference value to assign for any church	0.7	0.08
Additional scattering elements (decorations, thin columns, and roof)	+0.7	+0.08
Additional volumes between source and receiver as transept	+0.7	+0.08
Additional scattering/coupling elements such as big pillars or chapels along walls	+0.7	+0.08
Very high vaults or domes on tambour	+0.7	+0.08

proach and provide a simpler instrument to define typological values for both μ and k, the classification given in Table VI is finally proposed.

It can be observed that a new "reference" category was included to account for very simple spaces, which nonetheless cannot be considered sufficiently "proportionate" to fit Barron and Lee's model.[3] Groups of additional architectural features are listed, each corresponding to an incremental value of 0.7 for k and 0.08 for μ. In this way each church may assume k values from 0.7, for auditorium-like shapes, to 3.5, which should be considered only for very complex and large rooms with many additional scattering elements. This classification is particularly interesting, because it allows the rapid definition of the value of the typological parameter (be it k or μ). Simple volumes get a very low value, while complex spaces get larger values with increasing complexity. It is interesting to observe that the original typological classification given in Table V can be easily obtained by direct application of the incremental approach. In addition, the usefulness of the incremental approach can be observed, for example, in assigning the correct value to the church of St. Luca and Martina, which was difficult to assess according to Table V. In fact, starting from the reference value (0.08) and taking into account the presence of scattering elements due to decorations (+0.08), the transept braces between source and receivers (+0.08), and the dome (+0.08), the resulting μ value is 0.32, in good agreement with the empirical value.

Table VII shows that according to the new classification, the agreement with experimental values improved for some churches while it worsened for others, with substantially negligible differences (the largest rms variations being 0.08 dB for G, 0.09 dB for C_{80}, and 3.8 ms for T_S), so that the average absolute difference was substantially unchanged. This proves that the original classification and the new incremental classification are equivalent in terms of accuracy of results. So, the incremental approach is to be preferred if only for ease of use and for providing a unified selection criterion shared with the μ model.

V. COMPARISON BETWEEN ENERGETIC MODELS

The models investigated in this study together with the proposed typological classification were finally validated using the second sub-set of 12 churches. The performance of the revised model,[3] the modified double-rate model, and the μ model were compared by means of rms errors between measured and predicted values of G, C_{80}, and T_S.

Table VIII reports the parameters μ and k, resulting from application of the incremental typological classification, together with the parameter s required by the modified model and depending on the scattering characteristics of the chancel where the sound sources are located. In order to clarify the reasoning behind the assignment of the typological values of μ and k and, at the same time, provide a useful example of the way in which the incremental classification should be applied, Table VIII also reports the "sum" of the architectural features that individually contributed to the final rating.

As stated above, churches are complex buildings and assigning typological values may be a difficult task in which subjective factors (architectural and acoustic background, detailed knowledge of the building, and so on) may somewhat influence the final result. In order to understand how subjective factors may influence the choice of the input parameters μ, k, and s, five students with good architectural background (but limited acoustic knowledge) were asked to assign parameter values starting from the plans reported in Fig. 2 and

TABLE VII. Summary of experimental k values for each church surveyed, together with those assigned according to the old and the new classifications and absolute differences from experimental values. The variations in rms error resulting from application of the new classification, with reference to prediction of G, C_{80}, and T_S, are also reported for each church.

Church	k exp	Old k	New k	Δrms G	Δrms C_{80}	Δrms T_S
SSA	1.45	1.0	1.4	0.08	−0.09	3.80
SAP	1.24	1.0	1.4	0.07	−0.06	−1.71
MOD	1.90	2.0	2.1	0.01	−0.02	0.29
SNI	2.00	2.0	2.1	0.01	−0.03	−1.00
LUC	3.54	3.0	2.8	−0.04	0.04	0.64
SPB	2.84	3.0	3.5	−0.04	0.05	−1.25
BLA	3.25	2.0	2.8	−0.03	−0.00	0.26
JES	2.97	3.0	2.8	−0.00	−0.02	−2.36
SLM	3.30	3.0	2.8	0.03	0.02	−0.42
SMA	2.99	3.0	2.8	−0.03	0.03	0.16
CCT	3.00	3.0	2.8	0.07	−0.01	2.59
RIO	1.48	1.0	1.4	−0.02	0.01	−0.07
Mean absolute difference				0.04	0.03	1.21

TABLE VIII. Values of the coefficient derived for an application of the μ and modified models to the second set of churches. C=columns (scattering), Ch=chapels, T=transept, S=other scattering elements, and D =dome/vaults.

Church	Typological features	μ typology	μ calculation	k typology	k calculation	s typology
SMM	C	0.16	0.20	1.4	1.3	0.2
SPX	C	0.16	0.16	1.4	1.1	0.2
SAB	C+Ch	0.24	0.20	2.1	2.0	0.4
BAC	C+T	0.24	0.29	2.1	1.5	0.4
ACC	C+Ch+T	0.32	0.29	2.8	2.7	0.4
ORV	C+Ch+T	0.32	0.38	2.8	2.3	0.4
TOD	T+D	0.24	0.31	2.1	1.7	0.2
SAD	Ch+T+D	0.32	0.28	2.8	3.1	0.6
SAA	S+T+D	0.32	0.19	2.8	2.1	0.2
SLO	C+S+D	0.32	0.19	2.8	2.3	0.6
SMC	S	0.16	0.13	1.4	1.8	0.4
SGR	S	0.16	0.06	1.4	1.3	0.2

the classification reported in Tables I and VI. Median values corresponded to the values reported in Table VIII even though different evaluations were observed in a number of cases. Anyway, for all the subjects, the "error" in the typological choice was circumscribed to one incremental class above or below the median value. The inaccuracies resulting from errors in the choice of input parameters are discussed in Sec. V D.

The comparison between measured and predicted values of the three acoustic parameters (Fig. 3) was limited to receivers located in unobstructed areas in order to prevent the possibility that both direct sound and early reflections might be hindered, thus determining abnormal variations in the measured parameters that could arduously be accounted for by the predictive models.

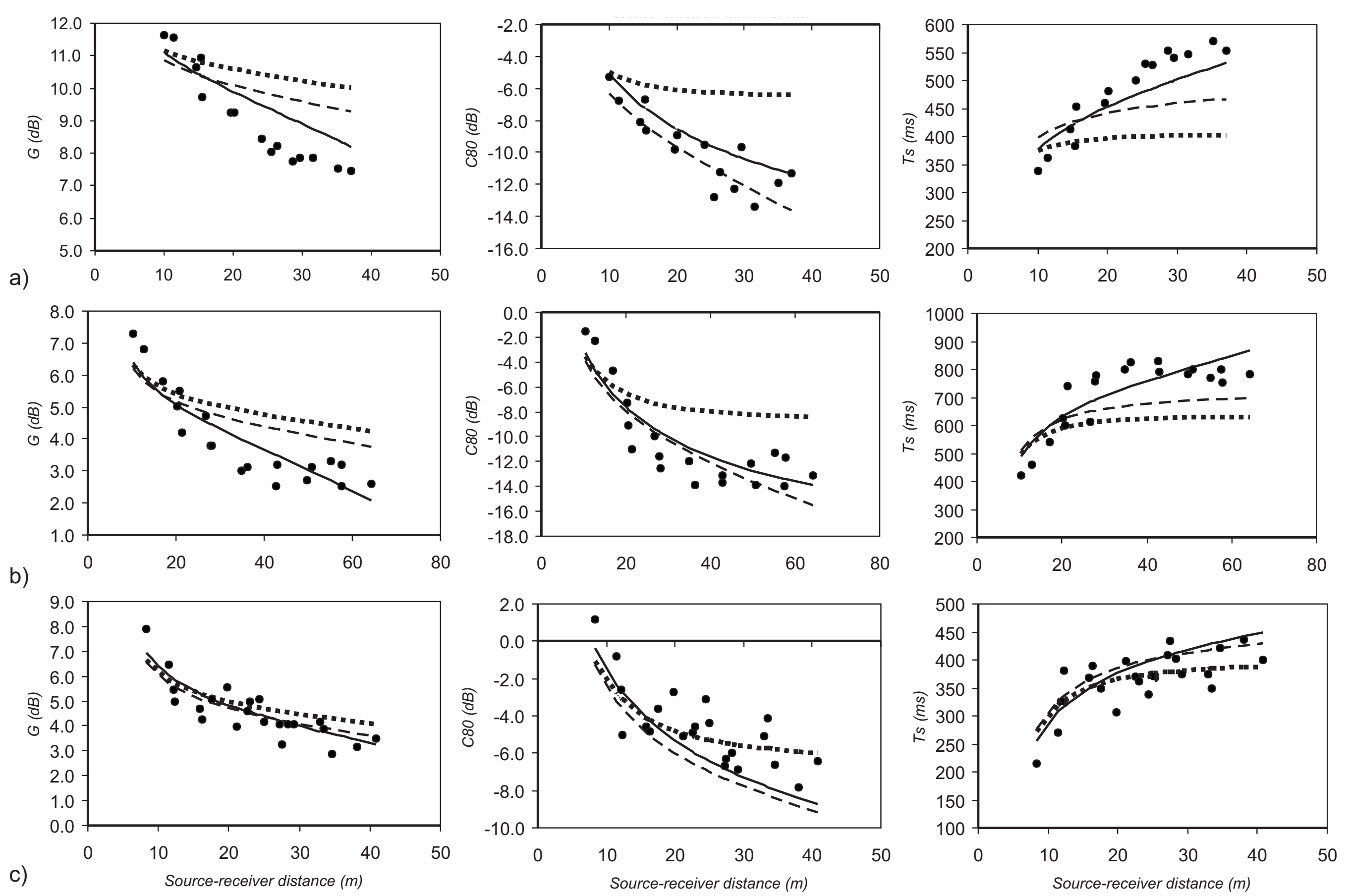

FIG. 3. Plot of measured (●) and predicted values of the strength (left), clarity (center), and center time (right) index at 1 kHz vs source-receiver distance according to the three models considered, revised theory (…), μ model (- - -), and modified theory (—). (a) ACC, (b) SAD, (c) SGR.

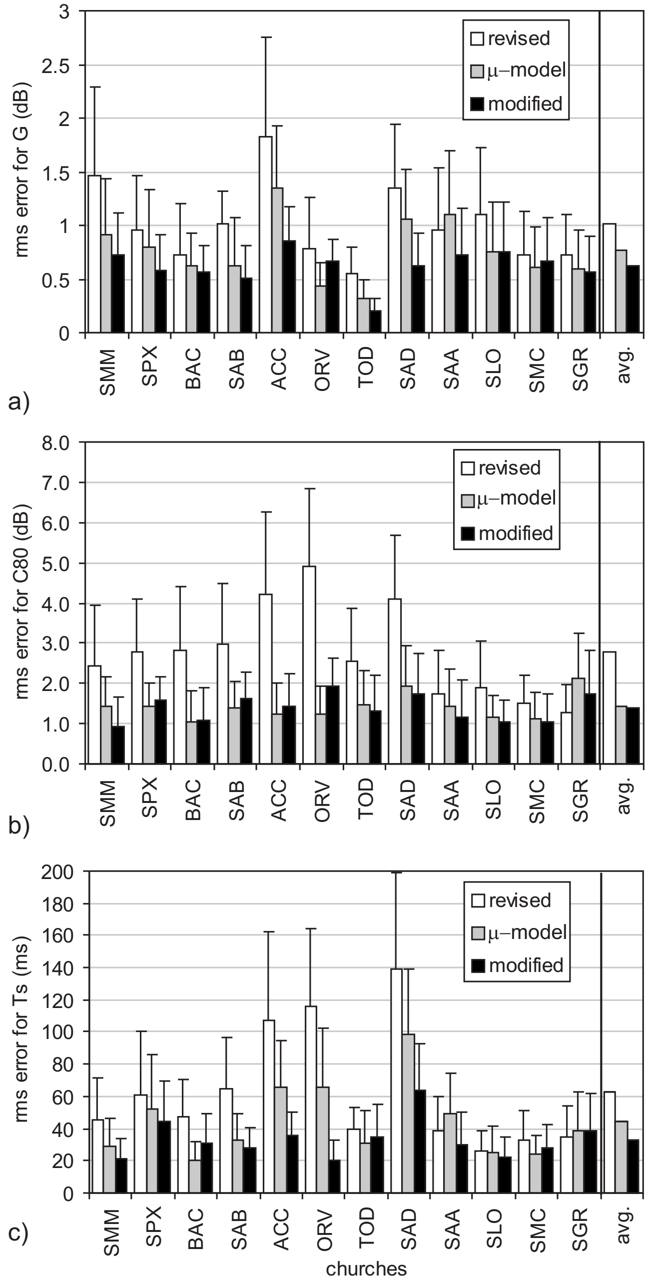

FIG. 4. rms error between measured and theoretical values of sound strength (a), clarity (b), and center time (c), at 1 kHz frequency band: theoretical values calculated with revised model, μ model, and modified model with the new classification. Error bars represent the standard deviation of point-to-point differences between measured and predicted values.

A. Sound strength

Among the acoustic parameters taken into account, sound strength shows the smallest variations in churches, possibly because it is largely influenced by long reverberation rather than by early reflections. As a consequence, this parameter might be predicted with reasonable accuracy even using the revised theory. However, as shown in Fig. 4(a), using either the modified or the μ model the prediction accuracy may be further improved. In fact, the average rms error varied from 1.01 dB for the revised model, to 0.77 dB for the μ model, to the smallest error of 0.62 dB obtained with the modified model.

The largest error was observed in Chiaravalle Abbey [Fig. 3(a)], where measured G showed a steep decrease as a function of distance (probably because of a lack of early reflections suppressed by the large transept), which was predicted with reliable accuracy only by the modified model. Similarly, the steep decrease observed in Sant'Andrea in Mantua [Fig. 3(b)] was better predicted by the modified model. The smallest errors were observed in Santa Maria della Consolazione where, probably because of the relatively small dimensions, the variation as a function of the distance was less evident.

B. Clarity

Comparisons between measured and predicted values of clarity were particularly interesting because the μ model was initially defined by calculating the μ value that provided the best agreement between predicted and measured data for clarity. Conversely, thanks to the proposed typological classification, μ values were then assigned, possibly leading to a loss in accuracy. Figure 4(b) shows that the μ model and modified model behaved the same, with an average rms error of 1.4 dB, nearly halving the error resulting from application of the revised theory (about 2.8 dB). Taking into account the performance for each church, it can be observed that in some cases the μ model provided better accuracy, while in other cases the modified model performed better, so that on average there was a substantial balance. It is interesting to observe that in Padre Pio Pilgrimage church in San Giovanni Rotondo [Fig. 3(c)] both models underpredicted clarity, while the revised theory provided the lowest error. This might depend on the relatively low ceiling that is likely to reflect early energy within the first 80 ms in a manner that is more similar to concert halls and auditoriums.

In some cases, such as Orvieto, Chiaravalle, and Mantua, the improvement resulting from using the μ or modified models was impressive, leading to errors that were one-third of those resulting from the revised theory.

C. Center time

Center time describes the temporal distribution of the sound energy without being affected by clarity drawbacks due to the arbitrary choice of a time limit to discriminate between useful and detrimental reflections. As a consequence T_S shows a distribution as a function of source-receiver distance, which is generally less scattered compared to C_{80} and, consequently, is more interesting to compare with values predicted by means of formulas. In addition, as the μ model is calibrated on clarity values (and its prediction accuracy is expected to be good for clarity), it is particularly interesting to validate its performance on a different parameter describing the same subjective attribute.

Figure 4(c) shows that on average, the modified model made it possible to nearly halve the rms error resulting from application of the revised theory (from 66 to 33 ms), while the μ model was in between (44 ms). Analysis of individual results shows that, as observed for clarity and, reasonably, for the same reasons, the Padre Pio Pilgrimage church in San Giovanni Rotondo [Fig. 3(c)] was the only one where revised theory is slightly more accurate (by 5 ms) than the

TABLE IX. rms error between measured and predicted values of sound strength, clarity, and center time, at frequency bands from 125 Hz to 4 kHz. Predicted values calculated with revised model, modified model, and μ model.

	rms error G (dB)			rms error C_{80} (dB)			rms error T_S (ms)		
Frequency	Revised	μ model	Modified	Revised	μ model	Modified	Revised	μ model	Modified
125	1.35	1.42	1.36	2.64	2.84	2.48	73	65	65
250	0.98	0.96	0.91	2.28	2.28	2.15	63	53	56
500	0.93	0.77	0.69	3.06	1.64	1.76	72	50	41
1000	1.01	0.77	0.62	2.77	1.42	1.39	63	45	33
2000	1.27	1.07	0.89	2.33	2.29	1.60	55	46	46
4000	1.87	1.33	1.05	1.89	3.47	1.79	36	43	43

other models. Among the other churches it is interesting to observe that the three churches that had already shown a significant improvement in prediction accuracy for C_{80} when the modified and μ models were used also confirmed the improvement for T_S. However, in this case the modified model provides better predictions than the μ model, probably because the latter affects only the first 80 ms, while in large churches weak early reflections are likely to arrive well beyond that time limit. Nonetheless, the modified model behaved quite badly in Mantua, with a rms error of about 66 ms (well above the average), probably as a consequence of the unusual decrease in values measured at the farthest points, possibly as a consequence of strong reflections from the back wall.

D. Frequency dependency and error analysis

In order to analyze the prediction accuracy over the whole spectrum, rms errors were calculated using the same parameter values defined at 1 kHz and assessing the difference between measured and predicted values over octave bands from 125 to 4000 Hz (Table IX). It can be observed that the models confirm the trends shown at 1 kHz even though larger errors appear at the extreme bands, where revised theory provides comparable levels of accuracy.

In particular, at 125 Hz all the models predict G values with errors above 1 dB, with revised theory performing slightly better than the others. From 250 to 4000 Hz the modified model provides the best results with errors below 1 dB except at 4 kHz.

When predicting C_{80} and T_S all the models perform badly at frequencies below 500 Hz, with larger errors than those observed at 1 kHz. From 500 to 4000 Hz modified model provides errors slightly above those observed at 1 kHz. The μ model behaves similarly when predicting T_S, but it becomes quite inaccurate when predicting C_{80} above 1 kHz; in fact, the rms error is 3.5 dB at 4 kHz. These larger errors are intrinsically due to the reduction in reverberation time typically observed at higher frequencies. Consequently, as the early reflected energy corresponds to the energy arriving $\mu/0.04$ times later, it may be significantly reduced when reverberation time gets shorter.

A further validation of the model performance was carried out by calculating the variation in prediction accuracy following an incorrect choice of the input parameters. Assuming that input parameters μ and k are badly selected by, at most, one "typological" class (i.e., respectively, by ± 0.08 and ± 0.7), the corresponding variations in the rms error are reported in Table X. Results show a general increase in the rms error, even though variations are small enough to allow both models to perform better than revised theory. The largest differences are observed for the μ model with reference to C_{80}. Figure 5 compares both models including the combined effect of the error in selecting k and s values. In general both models show larger differences as the source-receiver distance grows, with the highest variation observed when predicting C_{80} using the μ model. The error resulting from bad selection of s values is smaller than the error due to k and is negligible when k is underestimated, and a bit larger when k is overestimated.

VI. CONCLUSIONS

The results of a widespread acoustic survey taking into account a large number of Italian churches have been used to investigate and compare the prediction accuracy of energetic models in this kind of room. The whole sample of 24 churches was split into two halves in order to use the first group to define models and the other for their validation. In particular, the μ model, which was originally defined with reference to Mudejar-Gothic churches, was first generalized by showing that church-specific μ values could be grouped

TABLE X. rms average of the difference between errors corresponding to μ and k values assigned in Table VIII and errors obtained when typological class is badly selected. Upper class corresponds to $k+0.08$ and $\mu+0.7$, whereas lower class corresponds to $k-0.08$ and $\mu-0.7$. All values refer to at 1 kHz octave band.

	rms error G (dB)		rms error C_{80} (dB)		rms error T_S (ms)	
Church	Modified	μ model	Modified	μ model	Modified	μ model
Upper class	0.17	0.09	0.14	0.59	9.7	5.3
Lower class	0.22	0.16	0.25	0.51	7.9	8.6

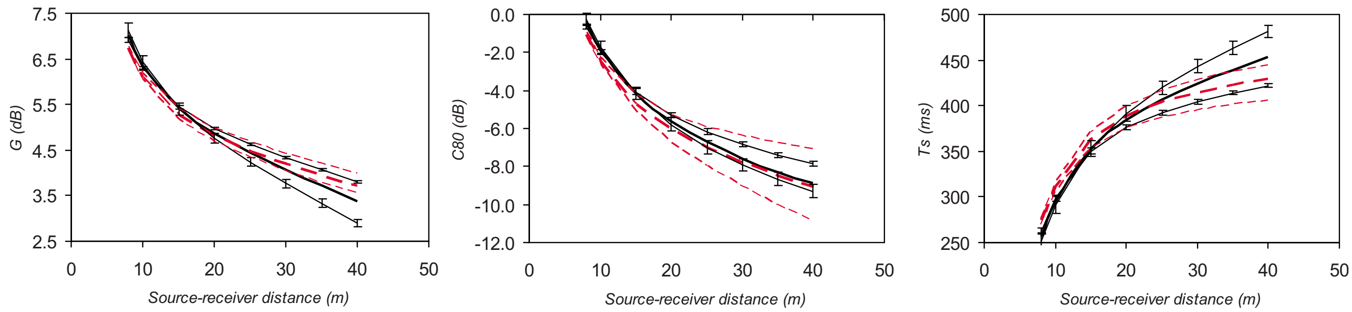

FIG. 5. (Color online) Plot of variations due to bad parameter estimation calculated for strength (left), clarity (center), and center time (right) index at 1 kHz as a function of source-receiver distance according to μ model (- - -) and modified theory (—). Thicker curves correspond to reference values calculated assuming $V=50\,000$ m^3, $T=5.5$ s, $S=15\,600$ m^2, $k=1.4$, $s=0.4$, and $\mu=0.16$. Thinner curves correspond to $k\pm0.7$ and $\mu\pm0.08$. Error bars represent variations corresponding to $s\pm0.2$.

according to typological characteristics. A typological classification was proposed using an incremental approach, so that any given architectural feature determines a corresponding increment of μ. This approach was then extended to the assignment of the parameter k for the modified theory, without significantly affecting its prediction accuracy.

The second half of the sample was first used to give an example of how the typological classification (valid for both the modified and μ models) should be applied as a function of architectural features. Then, the revised, modified, and μ models were compared in terms of their accuracy in predicting G, C_{80}, and T_S at 1 kHz. The results showed that strength could be predicted with reasonable accuracy (average rms error of 1.01 dB) even using the revised theory. However, the best results (average rms error of 0.62 dB) were obtained when the modified theory was used. Clarity showed larger variations as a function of source-receiver distance; consequently the revised theory gave on average larger rms errors of about 2.8 dB, while both the modified and μ model gave approximately the same rms error of 1.4 dB. Similarly, when dealing with center time, the modified model halved the error resulting from application of the revised theory (from 66 to 33 ms), while the μ model gave intermediate results (average rms error of 44 ms). Extension to other frequency bands showed a worse performance below 500 Hz, while at high frequencies the modified model provided consistent predictions (with slightly higher errors) for all the parameters. The μ model performed similarly for both G and T_S, while for C_{80} it showed much larger errors.

In conclusion, it can be stated that both models based on typological classification allowed an improvement in prediction accuracy compared to the revised theory. Among them, the μ model gave slightly less accurate predictions but required fewer input data (namely V, T, and μ), while the modified model gave the highest accuracy for all the acoustic parameters taken into account but required more parameters to be known (V, S, and T) and assigned (s and k). This led us to conclude that the choice of the model to use may be made according to the desired level of accuracy or, conversely, on the availability of the required input data, preferring the μ model when either less information is available or lower accuracy is desired, and the modified model when more information is available and the highest accuracy is desired. Such conclusions apply to midfrequencies, while at high frequencies the μ model should be used very carefully due to large errors in predicting C_{80}.

Further studies should be carried out in order to assess the possibility of extending model usage to different kinds of disproportionate rooms for which the revised theory cannot be reliably applied.

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