Predicting reverberation times in a simulated classroom

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(Received 3 March 2000; revised 9 July 2000; accepted 17 July 2000)

By varying the sound-absorption treatments in a simulated classroom, experimental results were compared with analytical and computer predictions of reverberation time. Analytical predictions were made with different absorption exponents, which are the result of different weighting procedures involving room surface areas and the sound-absorption coefficients. Sound scattering was found to influence measured reverberation times. With the amount of sound scattering provided, more accurate analytical predictions were obtained with absorption exponents that give reverberation times longer than those obtained with the Sabine absorption exponent, which consistently underpredicted reverberation times. However, none of the absorption exponents could be singled out as more adequate because of similar average accuracy. Computer predictions of reverberation time were accomplished with two commercially available ray-based programs, RAYNOISE 3.0 and ODEON 2.6, with specular and calibrated diffuse reflection procedures. Neither type of procedure, in either program, was more accurate than the best analytical predictions. With RAYNOISE, neither the specular nor the calibrated diffuse reflection procedure could be singled out as more adequate. For ODEON, the calibrated diffuse reflection procedure gave consistently more accurate predictions than its specular reflection procedure, with the best accuracy of the computer predictions. [S0001-4966(00)04710-X]

PACS numbers: 43.55.Br, 43.55.Dt, 43.55.Fw, 43.55.Ka [JDQ]

I. INTRODUCTION

It is often required to predict reverberation times in rooms for speech communication such as school classrooms. This paper compares the ability of several analytical expressions and two room acoustics computer programs to predict reverberation times in a simulated classroom with varied absorptive treatments.

The Sabine and Eyring reverberation formulas are most commonly used to predict reverberation time. These formulas are slightly different, because they are derived from somewhat different considerations,[1,2] but both are based on the assumption of a diffuse sound field. For high total sound absorption, the Sabine formula gives longer reverberation times than the Eyring formula, but the differences become smaller as the total amount of sound absorption decreases.

Many other formulas have been proposed for predicting reverberation times.[3–7] The development of some of these formulas was motivated by the lack of accuracy in reverberation time prediction when using the traditional Sabine/Eyring reverberation formula, in certain rooms with nonuniform surface absorption. This is a very important issue for many rooms, including classrooms, where the sound absorption is typically applied only to the ceiling area.

As far as having sound absorption located mostly on a single surface is concerned, classrooms are very similar to auditoriums because of the high audience absorption on the floor area in this type of room. A fundamental difference, however, is that recommended reverberation times for classrooms are well below 1 s,[8] whereas in larger rooms, such as opera houses and concert halls, values well above 1 s are usually recommended.[9]

Over the last three decades many room acoustical computer programs have been developed and used for predicting room acoustics quantities. These programs can be classified as wave-based programs and ray-based programs. Ray-based programs are the most common type of room acoustic programs available today.

The main objective of the present work was to systematically study the accuracy of seven reverberation formulas and two contemporary ray-based programs, RAYNOISE 3.0[10] and ODEON 2.6,[11] to predict reverberation times in a simulated classroom for varied absorption treatments. Another objective was to compare the effect of different absorption treatments to achieve recommended reverberation times in classrooms.

II. REVERBERATION TIME FORMULAS

All reverberation time formulas that have been used in the present work reduce to the form given, in SI units, by

$$T = 0.161 \frac{V}{Sa + 4mV}, \tag{1}$$

where V and S are the volume and the total surface area of the room, respectively, m is the sound attenuation constant of the air, and a is the so-called absorption exponent.

Different absorption exponents have been proposed. As we shall see, these are in fact the result of different weighting procedures involving the areas S_i of each of the room surfaces and the corresponding absorption coefficients α_i.

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Sabine[1] considered the absorption exponent as the average absorption coefficient $\overline{\alpha}$, given by

$$\overline{\alpha}=\frac{1}{S}\sum_{i}\alpha_i S_i\,, \tag{2}$$

where $S=\Sigma_i S_i$. Hence, according to Sabine, the absorption exponent is

$$a_{\mathrm{Sab.}}=\overline{\alpha}. \tag{3}$$

Equation (1), with $a=a_{\mathrm{Sab.}}$, is known as the Sabine reverberation formula.

Eyring[2] was concerned with the fact that when $a_{\mathrm{Sab.}}=1$, that is, for the case where the average absorption coefficient $\overline{\alpha}$ is unity, the reverberation time does not become zero. Eyring proposed a reverberation formula in which the absorption exponent is calculated according to

$$a_{\mathrm{Eyr.}}=-\ln(1-\overline{\alpha}). \tag{4}$$

The Eyring reverberation formula—Eq. (1) with $a=a_{\mathrm{Eyr.}}$—gives reverberation time equal to zero for $\overline{\alpha}=1$. It reduces to the Sabine formula for $\overline{\alpha}\ll 1$.

Millington[3] was concerned with the fact that when the absorption coefficients of highly absorbing materials are measured, the Eyring formula gives absorption coefficients greater than unity. Millington then developed a reverberation formula, which when used for the calculation of the absorption coefficient of samples in reverberation chambers, always results in sample absorption coefficients less than unity. The Millington formula is given by Eq. (1) with the absorption exponent given by

$$a_{\mathrm{Mil.}}=-\frac{1}{S}\sum_{i}S_i\ln(1-\alpha_i). \tag{5}$$

The Millington formula has the drawback that when one of the surfaces of the room, even if very small, has an absorption coefficient $\alpha_i=1$, $a_{\mathrm{Mil.}}$ would be infinitely large and hence the reverberation time would be zero. This happens because, as mentioned above, absorption coefficients obtained using the Millington formula are always less than unity. Therefore, the traditional absorption coefficients obtained in the reverberation chamber using the Sabine formula cannot be used in the $a_{\mathrm{Mil.}}$ formula as given by Eq. (5). To enable the Millington formula to be used correctly, Dance and Shield[12] have created a conversion graph, so that Millington absorption coefficients can simply be estimated from the standard absorption coefficients.

The fundamental difference between the Eyring and Millington approaches is that the former considers the energy to be uniformly spread out after each reflection, whereas the latter considers the acoustical energy in a series of confined sound cones, reflected in sequence by each of the room surfaces S_i. A recommendation by Cremer and Müller[4] consists of dividing the total room surface area S into several large ‘‘principal surfaces,’’ which can be regarded as encountered by the sound cones in sequence (Millington’s approach), and to subdivide these principal surfaces into smaller surfaces, which can be regarded as being uniformly acoustically irradiated (Eyring’s approach).

The smaller subdivisions in each of the principal surfaces are first averaged according to Eq. (2), to determine the mean absorption coefficient for each of the principal surfaces. These are then inserted into Eq. (5) giving Cremer’s absorption exponent as

$$a_{\mathrm{Cre.}}=\frac{1}{S}\sum_{i}S_i\left[-\ln\left(1-\frac{1}{S_i}\sum_{j}\alpha_{ij}S_{ij}\right)\right]. \tag{6}$$

S_{ij} is the surface area of each subdivision j, of the principal surface i, which has the absorption coefficient α_{ij}. In the case where the principal surfaces have a uniform absorption coefficient, $\alpha_{ij}=\alpha_i$, and $a_{\mathrm{Cre.}}=a_{\mathrm{Mil.}}$. For that reason we have used Eq. (6) with the Millington absorption coefficients. This combined formula embraces both the Eyring and Millington formulas where they are adequate, but avoids their physically impossible results.

Because nonuniform distribution of absorption in rooms often occurs in practice, the condition of a diffuse sound field is frequently not fulfilled. Kuttruff[13] obtained sound decay curves at different points in a rectangular room, with nonuniform surface absorption but with surfaces that reflect energy in an ideally diffuse way (Lambertian scattering), by numerically solving an integral equation. He found that the initial decay is characterized by fluctuations. After the time that it takes for the sound to travel a few mean free-path lengths, these initial fluctuations fade out, leaving an exponential decay with the same decay constant throughout the whole room. He then numerically calculated absorption exponents, in cubic and rectangular rooms, with different distributions of surface absorption.

In another related study, Kuttruff[5,9] proposed a correction to the Eyring absorption exponent, to take into account the influence of nonuniform surface absorption in the room. An additional correction factor was added to $a_{\mathrm{Eyr.}}$, that takes into account the influence of unequal path lengths. This correction is based on the variance of the path length distribution γ^2, which is given by $\gamma^2=(\overline{l^2}-\overline{l}^2)/\overline{l}^2$. $\overline{l}$ is the mean free path given by $\overline{l}=4V/S$, and $\overline{l^2}$ is the mean squared value of the free paths between two subsequent wall reflections. When both corrections are combined, Kuttruff’s absorption exponent is given by

$$a_{\mathrm{Kut.}}=a_{\mathrm{Eyr.}}\left(1-\frac{\gamma^2}{2}a_{\mathrm{Eyr.}}\right)+\frac{\Sigma_i(1-\alpha_i)(\overline{\alpha}-\alpha_i)S_i^2}{S^2(1-\overline{\alpha})^2}. \tag{7}$$

The first term in Eq. (7) reflects the influence of unequal path lengths, and the second, the nonuniform surface absorption. For rectangular room shapes, such as classrooms, γ^2 is close to 0.4.[9]

Fitzroy[6] experimentally verified, in rooms where the absorption is nonuniformly distributed, that the Sabine and Eyring reverberation formulas give reverberation time predictions that usually ‘‘vary widely’’ from measurements. According to his experience these formulas underpredicted reverberation times, especially in rooms that were heavily damped in the vertical direction. This is the case for rooms with an acoustical ceiling, as typically found in classrooms, or with high audience absorption as found in auditoriums. He then proposed a reverberation formula in which the absorp-

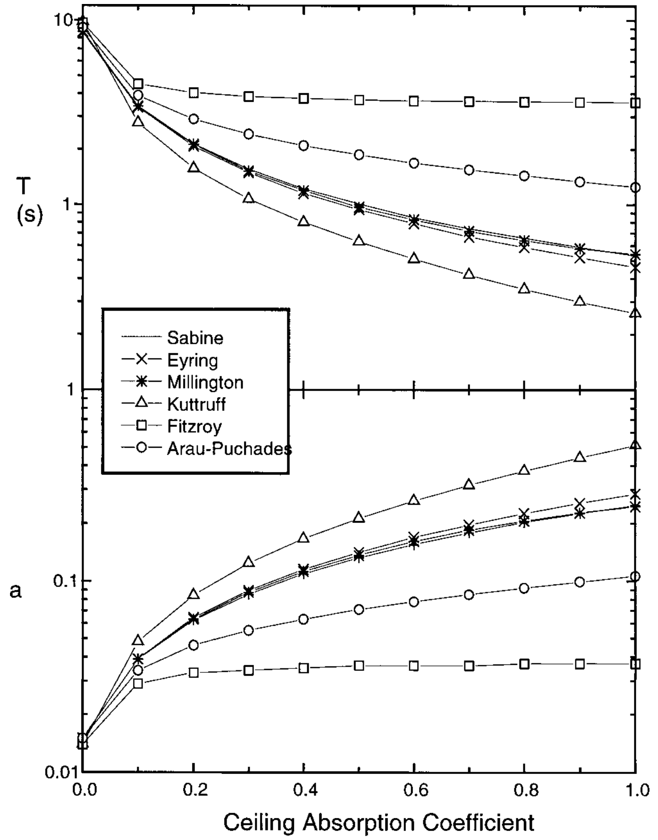

FIG. 1. Reverberation times (upper plot) and values of the absorption exponents (lower plot) versus the ceiling absorption coefficient. These results are for the room used to simulate the classroom, in which sound absorption is applied to the ceiling area.

tion exponent is calculated by an area-weighted arithmetic mean of an Eyring-type absorption exponent in the three orthogonal directions. Fitzroy's absorption exponent is given by

$$a_{\text{Fit.}}=-S\left[\frac{S_x}{\ln(1-\overline{\alpha}_x)}+\frac{S_y}{\ln(1-\overline{\alpha}_y)}+\frac{S_z}{\ln(1-\overline{\alpha}_z)}\right]^{-1}. \quad (8)$$

In Eq. (8), S_x is the ceiling plus the floor surface area, S_y is the surface area of both side walls, and S_z is the surface area of both end walls. Here α_x, α_y, and α_z are the mean absorption coefficients of the surface areas S_x, S_y, and S_z, respectively, which are calculated according to Eq. (2).

Based on Fitzroy's idea, Arau-Puchades[7] has proposed a reverberation formula in which the absorption exponent is given by weighting an Eyring-type absorption exponent in each one of the main directions according to

$$a_{\text{ArP.}}=[-\ln(1-\overline{\alpha}_x)]^{S_x/S}\cdot[-\ln(1-\overline{\alpha}_y)]^{S_y/S}\cdot[-\ln(1-\overline{\alpha}_z)]^{S_z/S}. \quad (9)$$

Arau-Puchades experimentally confirmed the adequacy of his absorption exponent in auditoriums, theaters, and television broadcasting studios.

Figure 1 shows values of the various absorption exponents and the corresponding reverberation times as functions of the ceiling absorption coefficient. The sound-absorbing configuration chosen for this comparison is typical of classrooms, in which sound absorption is applied to the ceiling area. For this type of sound-absorbing configuration, $a_{\text{Cre.}}$ is equal to $a_{\text{Mil.}}$. It can be seen that the absorption exponents of Sabine, Eyring, and Millington give reverberation times that are practically the same. This is because, even for the extreme case of a ceiling absorption coefficient equal to 1, the average sound-absorption coefficient $\overline{\alpha}$ is only equal to approximately 0.25, for this particular sound-absorbing configuration. This gives $a_{\text{Cre}}=a_{\text{Mil}}\approx a_{\text{Sab}}=0.25$, and $a_{\text{Eyr}}=0.29$. The absorption exponent of Arau-Puchades gives longer reverberation times, and that of Fitzroy even longer. The Kuttruff absorption exponent gives the shortest reverberation times compared to the other proposals. For the largest values of the ceiling absorption coefficient, Fig. 1 shows differences in reverberation time up to one order of magnitude. One of the objectives of the present study is to compare predictions of reverberation time by the various absorption exponents with measurements in a simulated classroom, for different sound-absorbing configurations.

III. EXPERIMENTAL PROCEDURES

A. The room and the sound-absorbing configurations

In the present study, the classroom was simulated in a rectangular and reverberant laboratory enclosure. The room is 9.20 m long by 4.67 m wide and 3.56 m high. Sound absorption in the simulated classroom was varied by laying different amounts of sound-absorbing ceiling tiles on the floor and walls of the room. The maximum amount of ceiling tiles used was 42.24 m^2. Figure 2 shows schematics depicting the application of the ceiling tiles in different amounts and configurations.

For ease of handling, the ceiling tiles were laid on the floor to simulate ceiling absorption. The untreated room, configuration (0) in Fig. 2, was tested first. Thereafter, the amount of sound-absorbing material was progressively increased, in different configurations, which corresponded to areas of 26.2%, 52.4%, 78.7%, and 98.3% of the total ceiling area of 42.96 m^2, and are referred to as configurations (25), (50), (75), and (100), respectively.

The amount of sound-absorbing material of configuration (50) (52.4% of the ceiling area) was also tested in different configurations to evaluate their effectiveness. In Fig. 2, configuration (HR) represents covering the ceiling on the receiver side; configuration (HS) represents covering the ceiling on the source side; configuration (EW) represents covering the end wall and part of the ceiling; configuration (PW) represents covering the upper part of the walls; and configuration (PF) represents covering a ring on the ceiling. All had exactly the same area of added sound-absorbing material corresponding to 52.4% of the complete ceiling area.

The sound-absorbing material used consisted of 25-mm-thick Luna Perforated Ceiling Tiles; they were semirigid glass-fiber panels (0.60×1.21 m) for ceiling applications. In tests conducted in a reverberation chamber according to ASTM C423, the following sound-absorption coefficients were obtained in the six-octave frequency bands from 125 Hz to 4 kHz: 0.08, 0.44, 0.94, 1.15, 1.01, and 0.75. The room surfaces are painted, nonporous masonry.

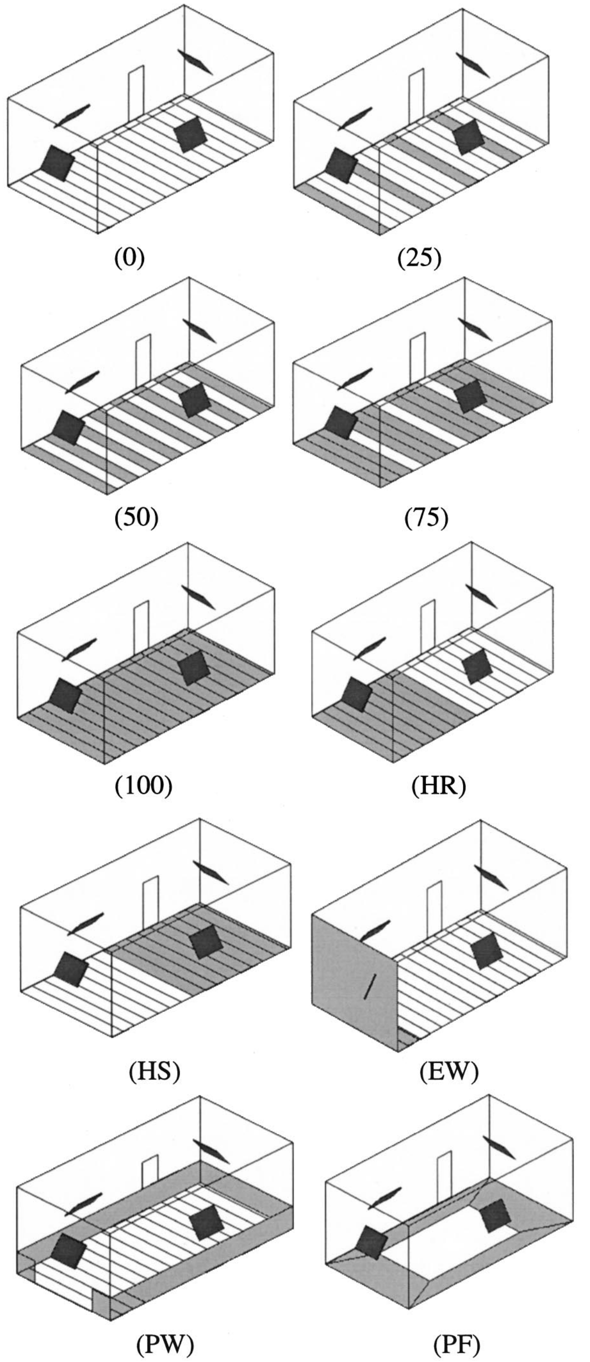

FIG. 2. Schematics depicting the application of the sound-absorbing ceiling tiles in different amounts and configurations. Configuration (0) depicts the room with no absorption. Configurations (25), (50), (75), and (100) depict the room with ceiling tiles applied, respectively, on 26.2%, 52.4%, 78.7%, and 98.3% of the floor area. Configurations (HR), (HS), (EW), (PW), and (PF) depict sound-absorbing configurations that have areas equal to 52.4% of the floor area. Also shown are the diffuser panels and the room door.

B. The measurements and the measuring system

For practical reasons, reverberation time measurements are in general conducted in unoccupied classrooms, because reverberation times in occupied classrooms would vary with the number of people in the classroom.

As will be discussed later, it was found necessary to add sound scattering to the room used to simulate the classroom. This was accomplished by fitting the room with diffuser panels, made of gypsum board (1.2×0.9 m). Figure 2 shows the room fitted with four diffuser panels, which were placed at an approximately 45° angle with each wall.

The temperature and relative humidity at the time of the tests were also measured to obtain the applicable air attenuation constant. The reverberation times measured in the bare room, configuration (0), were used to estimate the sound-absorption coefficients of the bare room surfaces using the Eyring formula.

In the results included here, the microphone was omnidirectional, and was located at a height of 1.1 m in six positions distributed where students would sit. The measured reverberation times reported here are given as position-averaged values. Two sound sources were used, one approximately omnidirectional and the other with an average directivity index at midfrequencies of 5 dB straight ahead, to better approximate the directivity of a human talker. The sources were positioned on the centerline of the room, 1.5 m from the front wall and the floor. Reverberation time measurements with the omni source were used in comparisons with the analytical predictions, and those with the directional source with the computer predictions. For these two sources, the average difference between measured reverberation times in all configurations was 4.1%.

Reverberation time measurements were accomplished with the RAMSOFT measuring system. This system uses a maximum-length-sequence (MLS) signal and a fast Hadamard transform procedure to obtain measured impulse responses at particular locations in rooms. A program filters the measured impulse responses into standard octave frequency bands and calculates decay times by means of Schroeder's backward integral. The validity of the measurement program was verified in various situations, including in an international round robin of room acoustics measurement systems.[14]

IV. RESULTS AND DISCUSSION

A. Effect of sound scattering on reverberation time measurements

Hodgson[15] investigated the effects of increasing sound scattering on the decay times and the stationary sound-pressure levels in a rectangular room. In his work, both quantities were predicted by ray-tracing simulations and by using the Eyring formula for diffuse sound fields. In the ray-tracing simulations, sound scattering was accomplished by two different types of scattering mechanisms: a Lambert model for surface scattering, and volume scatterers. He then found, that independent of the scattering mechanism used, as scattering increases, sound decays predicted by ray tracing tended to better approximate the results obtained using the Eyring reverberation formula.

Although surface and volume scattering have similar effects on the room sound field, they are quantified differently. Surface scattering is measured by the *scattering coefficient* δ, which is defined as the ratio of non-specularly reflected sound energy to totally reflected energy. Volume scattering is measured by the *scattering frequency* v [m^{-1}], obtained by multiplying the density of scatterers by the average scattering cross section.[16]

There have been some attempts to measure surface scattering,[17] and studies on the scattering of sound by fittings in industrial rooms.[16,18] However, these are still the objects of ongoing research. Surface and volume scattering are present during measurements in real classrooms. Unlike the absorption coefficient, the scattering coefficients of room surfaces are not available. Although in unoccupied classrooms volume scattering by the occupants is not an issue, the presence of desks and other furniture could be considered to be volume scatterers. These considerations point to the need of quantifying the amount of scattering in real classrooms. For obvious reasons, this was not possible. It was decided to experimentally verify the influence of volume scattering on measured reverberation times, and to report reverberation times based on a reference value for the scattering frequency.

A recommendation by Kuttruff[9] for achieving a diffuse sound field in reverberation chambers is to have the scattering frequency in the range $0.5/H < v < 2.0/H$, where H is the distance of the test specimen from the wall opposite to it. In the room used to simulate the classroom, this would give a range for the scattering frequencies of $0.141\ \mathrm{m}^{-1} < v < 0.562\ \mathrm{m}^{-1}$. The corresponding range for the number of diffuser panels $(1.2 \times 0.9\ \mathrm{m})$ N would be $40 < N < 160$.

However, the results of the computer simulations performed by Hodgson[15] revealed that a scattering frequency of $0.050\ \mathrm{m}^{-1}$ was sufficient for the sound decay to agree with that predicted by the Eyring formula. Benedetto *et al.*[19] investigated the effect of stationary diffuser panels in the measurement of sound-absorption coefficients in a reverberation chamber. They found experimentally that a scattering frequency of $0.024\ \mathrm{m}^{-1}$ was sufficient to obtain absorption coefficients practically equal to those obtained with a scattering frequency of $0.032\ \mathrm{m}^{-1}$. This means that increasing the scattering frequency above $0.024\ \mathrm{m}^{-1}$ had no effect on the degree of diffusion in the reverberation chamber. Based on these results, the number of diffuser panels $(1.2 \times 0.9\ \mathrm{m})$ necessary to achieve diffuse field conditions in the simulated classroom would be $N = 14$ according to the former study, and $N = 7$ according to the latter.

To get an idea of the influence of the number of panels on reverberation time, a limited number of reverberation time measurements were made in the room for some of the sound-absorbing configurations, and with the number of diffuser panels varying from zero to 12 panels. These would give scattering frequencies ranging from 0 to $0.042\ \mathrm{m}^{-1}$. It was then found that as the number of diffuser panels increase, the measured reverberation times decrease, and tend to better approximate predictions obtained using the Eyring formula.

The better agreement between measured and predicted reverberation times as the number of diffuser panels increase was expected, since increasing scattering also increases the randomization of the incidence of sound on the room surfaces, resulting in a sound field that is more diffuse. However, even with 12 diffuser panels, significant differences were still apparent between measured and predicted reverberation times. A natural course would be to keep increasing the number of diffuser panels beyond 12, to attempt to achieve a diffuse sound field, here indicated by the agreement between measured and predicted reverberation times. This, however, was not pursued because the objective was to reproduce acoustical conditions found in typical classrooms, which usually are not ideal diffuse sound fields. The creation of a diffuse sound field was not sought since it would represent an unrealistic approximation to unoccupied classrooms.

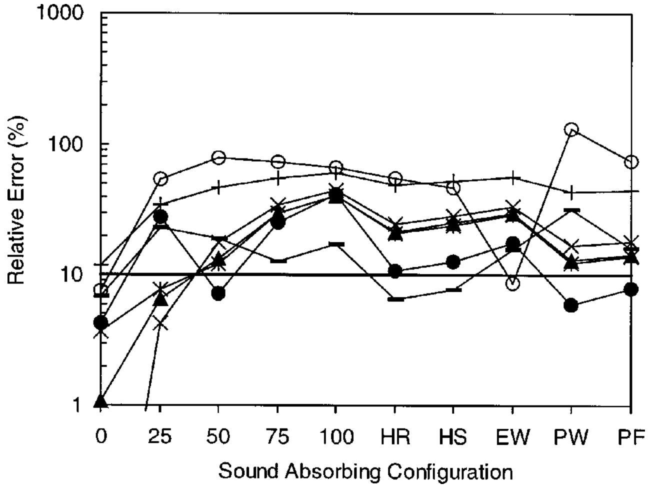

FIG. 3. Average relative error in reverberation time prediction across the six octave bands from 125 Hz to 4 kHz, for each sound-absorbing configuration. Analytical predictions using absorption exponents of: (▲) $a_{\mathrm{Sab.}}$, (×) $a_{\mathrm{Eyr.}}$, (*) $a_{\mathrm{Mil.}}$, (●) $a_{\mathrm{Cre.}}$, (+) $a_{\mathrm{Kut.}}$, (○) $a_{\mathrm{Fit.}}$, and (−) $a_{\mathrm{ArP.}}$. Also shown in this figure is a horizontal line across the plot area that corresponds to an accuracy of 10%.

According to both studies mentioned above,[15,19] a value for the scattering frequency of $0.012\ \mathrm{m}^{-1}$ was about the minimum necessary for the most pronounced changes to occur in the values of the parameters used to indicate the degree of sound diffusion. These parameters were reverberation time in the first study,[15] and the sound-absorption coefficient in the second.[19] Therefore, both studies seem to indicate that a scattering frequency of around $0.012\ \mathrm{m}^{-1}$ is a ''borderline'' value for the most significant changes in the sound field to occur. This was taken as a convenient reference value for the measurements in the simulated classroom. Therefore, all reverberation time measurements reported here were obtained with the room fitted with four diffuser panels $(1.2 \times 0.9\ \mathrm{m})$, giving a scattering frequency of $0.014\ \mathrm{m}^{-1}$.

B. Accuracy of reverberation time formulas

For each sound-absorbing configuration in the simulated classroom, Fig. 3 shows the average relative error in reverberation time prediction, across the six octave bands from 125 Hz to 4 kHz, using the different absorption exponents. Also shown in this figure is a horizontal line across the plot area that corresponds to an accuracy of 10%, which was adopted by Hodgson[20] as an engineering-type accuracy for reverberation time predictions in practical applications. Although a just-noticeable difference in reverberation time is about 5%,[21] the 10% accuracy is more indicative of a minimum practically important difference.

It can be seen in Fig. 3 that for the bare room [configuration (0)], all absorption exponents are capable of predicting reverberation time with an accuracy of 10%. This was expected since in the absence of ceiling tiles, all predictions for the bare room tend to approximate Eyring's prediction. As mentioned earlier, the absorption coefficients of the room surfaces were obtained from measurements in the bare room using the Eyring formula.

For configuration (25), the Sabine, Eyring, and Millington absorption exponents were all capable of predicting reverberation times with an accuracy of 10%; and for configurations (50), (PW), and (PF), only the Cremer absorption exponent predicted with this accuracy. For configuration (EW), only the Fitzroy absorption exponent was successful in predicting with an accuracy of 10%, and for configurations (HR) and (HS), only the Arau-Puchades absorption exponent predicted reverberation times with this accuracy. Predictions for configurations (75) and (100) could not be made with an accuracy of 10% by any absorption exponent. For these two configurations, predictions with the smallest average relative error were obtained with the Arau-Puchades absorption exponent, with values of 12.6% and 17.4%, respectively.

Figure 4 shows experimentally obtained reverberation times in octave bands, together with the prediction that gives the smallest average relative error for the specific sound-absorbing configuration (best prediction). Also shown in Fig. 4 are the predictions using the Sabine formula. As mentioned earlier, configuration (0) was used to estimate the absorption coefficients of the bare room surfaces using the Eyring formula. Therefore, for configuration (0), Fig. 4 shows that the average relative error using the Eyring formula is 0.0%.

It can be seen in Fig. 4 that the Sabine formula tends to underpredict reverberation times in the simulated classroom, with different amounts and configurations of sound absorption. Therefore, predictions obtained with lower values of the absorption exponent tend to be more accurate because they give longer reverberation times. For the sound-absorbing configurations tested, more accurate predictions than those obtained using the Sabine formula were obtained with the absorption exponent of Arau-Puchades in four configurations [(75), (100), (HR), (HS)]; of Cremer, in three configurations [(50), (PW), (PF)], and even with Fitzroy's absorption exponent in configuration (EW). Based on these results, the Arau-Puchades and Cremer absorption exponents better follow the measured changes in reverberation time with configuration.

To get an idea of the degree of uniformity of the predictions in different frequency bands, the average relative error at midfrequencies (500 Hz, 1 kHz, and 2 kHz) was also calculated for each sound-absorbing configuration, together with the relative errors in the 1-kHz frequency band. These relative errors, and the average relative errors across the six octave bands from 125 Hz to 4 kHz, were used to determine the ''overall quality'' of the predictions by averaging the relative errors for all configurations tested.

Table I shows the *overall average relative errors* in reverberation time predictions for each absorption exponent. This table shows that as more bands are included in the averages, the relative errors are reduced. In fact, the relative errors in the 1-kHz band are larger than the average relative errors at midfrequencies, which are larger than the average relative errors across the six octave bands from 125 Hz to 4 kHz. This result reveals that the quality of the predictions is nonuniform throughout the frequency range, and the average relative errors in reverberation time prediction tend to become smaller as more frequency bands are included in the averages.

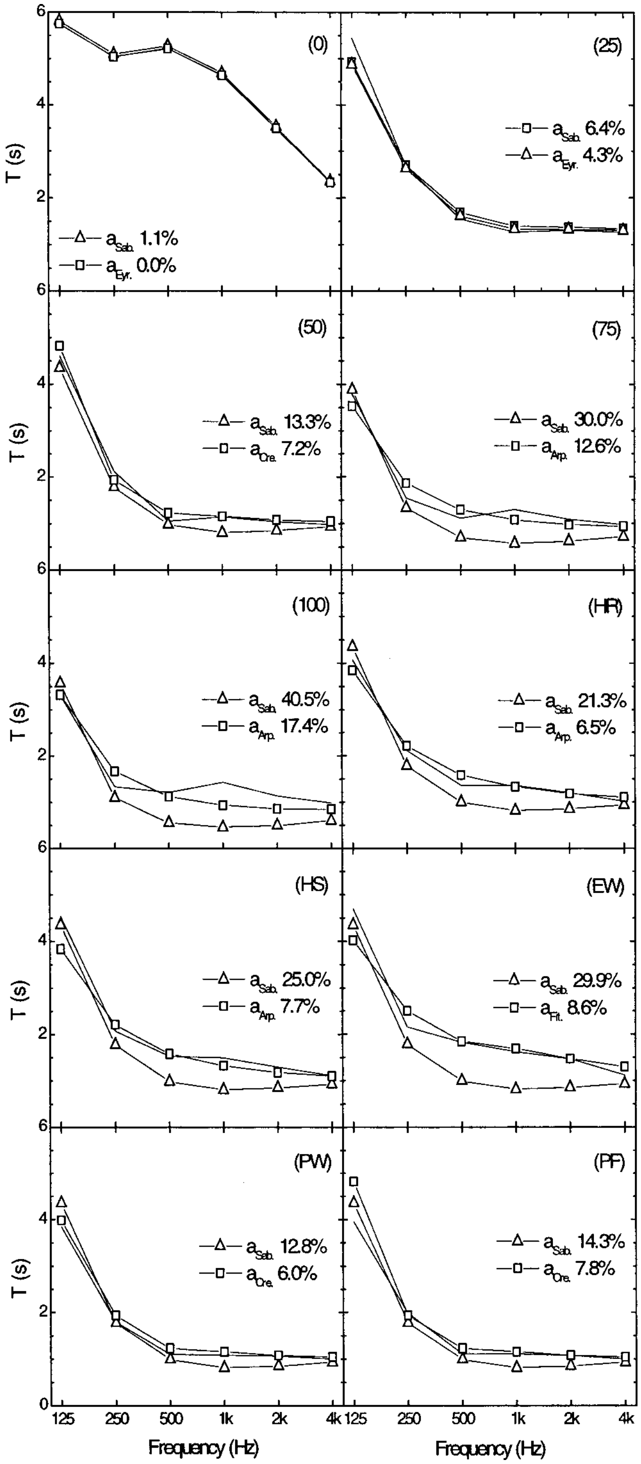

FIG. 4. (———) Experimental and analytical predictions of reverberation time. (△) predicted by the Sabine formula; (□) analytical prediction that gives the smallest average relative error (best prediction). Indicated in each configuration is the average relative error of the prediction across the six octave bands from 125 Hz to 4 kHz.

In Table I, by comparing the overall average relative

TABLE I. Overall average relative errors of the analytical predictions of reverberation time for ten sound-absorbing configurations in the simulated classroom.

	Overall average relative error (%)		
	Frequency bands included in the averages		
Absorption exponent	1 kHz	500 Hz–2 kHz	125 Hz–4 kHz
Sabine	38.8	31.6	21.5
Eyring	42.6	35.7	24.7
Millington	36.1	30.4	21.1
Cremer	26.7	23.9	17.4
Kuttruff	68.0	63.5	49.3
Fitzroy	92.0	95.8	65.7
Arau-Puchades	22.9	22.7	16.7

errors across the six octave bands from 125 Hz to 4 kHz, it can be seen that the Arau-Puchades and Cremer absorption exponents predict reverberation times with the smallest average relative errors of 16.7% and 17.4%, respectively. The largest average relative errors were obtained by predicting reverberation times with the Kuttruff and Fitzroy absorption exponents, with values of 49.3% and 65.7%, respectively.

Dance and Shield,[12] using the Sabine, Eyring, and Millington reverberation formulas, predicted reverberation times in a recording studio with average relative errors of 12.8%, 20.1%, and 10.9%, respectively. In a concert hall the average relative errors were 35.6%, 27.9%, and 36.6%, respectively. Table I shows that the average relative errors in reverberation time predictions using these formulas in the simulated classroom, fall more or less in the middle of both sets of average relative errors, with values of 21.5%, 24.7%, and 21.1%. However, a fundamental difference between both studies is that these reverberation formulas had consistently overpredicted reverberation times in the concert hall, while the results of the present study reveal that these same formulas have consistently underpredicted reverberation times in the simulated classroom.

Figure 4 shows that absorption exponents that give reverberation times longer than those predicted by using the Sabine absorption exponent would lead to more accurate predictions of conditions in the simulated classroom. However, Table I shows that the average relative errors produced by these absorption exponents are comparable to those produced using the Sabine and Eyring absorption exponents. The most accurate absorption exponents resulted in average relative errors in the range between 17% and 25%. Therefore, with the rather limited amount of experimental data available, it seems prudent not to single out any of these absorption exponents as more adequate because of similar average relative errors.

This reveals a considerable uncertainty in choosing a particular absorption exponent for predicting reverberation times for a given room condition. In other words, a given absorption exponent may predict the reverberation time accurately in one situation but fail in another. This is the main drawback of reporting average values. These results do not justify the need to use the more complex analytical expressions to predict reverberation times because they do not in general lead to greater accuracy. Because of its simplicity and average accuracy comparable to the best predictions, the Sabine/Eyring formula is concluded to be a reasonable choice among the analytical expressions compared here.

C. Accuracy of computer models

Both RAYNOISE and ODEON can be used with pure specular or with diffuse reflection procedures. Modeling surface reflections as partially diffuse, at least for the later reflections, is more realistic relative to actual conditions in rooms. It additionally provides greater flexibility for modeling, but at the same time introduces another variable that one must consider. However, data describing the diffusing properties of common room surfaces are simply nonexistent. Usually, in ray-based programs in which reflections are modeled as diffuse, the diffusion coefficient assigned to a given surface is treated as the probability that a given ray hitting the surface will be diffusely reflected. If the reflection happens to be diffusing instead of specular, the ideal Lambertian model determines the new ray direction and the intensity reaching the receivers.

Experience in dealing with diffuse reflections in ray-based computer programs is very limited. One approach is to start with a purely specular model, and to compare sound decays thus obtained with those derived from the classical Sabine/Eyring formula. It was found that in the case of lack of agreement, reverberation times predicted by a diffuse model could be made to approximately agree with those obtained from the classical reverberation formulas by arbitrary adjustments to the diffusion coefficients of various surfaces. This is in general easy to accomplish by a rather crude trial-and-error procedure, in which the surfaces that are assigned as diffuse reflectors and the respective diffusion coefficients are varied. However, no consistent general procedure could be established by analyzing the results thus obtained. Modeling reduces to an exercise of matching the computer results with those obtained from classical reverberation formulas. If one were to rely on these formulas to make the computer modeling successful, there would be no need for computer modeling, at least for predicting reverberation times.

The real advantage of computer models is to deal with nondiffuse fields where the classical Sabine formula may not be appropriate. Since these are to various degrees very common, an approach is required to model these rooms. The approach adopted by the present work was that of ‘‘calibrating’’ the diffuse model. The calibration procedure consisted of first modeling a room of ‘‘similar characteristics’’ for which the reverberation time was known, from measurements, for instance. If the specular model did not predict the measured reverberation time, then one hoped that a diffuse model could be found to approximate the expected results. The diffusion coefficients of the surfaces, considered as diffuse reflectors, were varied until predicted and measured reverberation times agreed within a certain accuracy. This approach seems realistic for classrooms because many classrooms have quite similar characteristics. However, it may not be applicable at the design stage of most types of new rooms, where a combination of scale models and computer models may be required to study the acoustical properties of the new room.

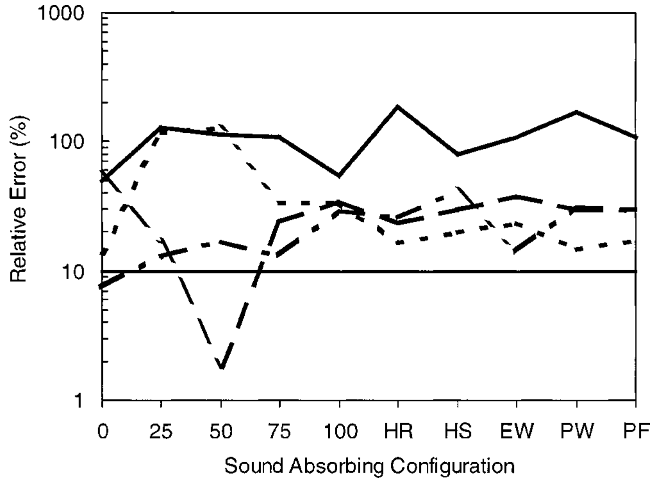

FIG. 5. Average relative errors in reverberation time predictions across the six octave bands from 125 Hz to 4 kHz, for each sound-absorbing configuration. Computer predictions using: (---) RAYNOISE specular model; (– –) RAYNOISE diffuse model; (———) ODEON specular model; and (– – –) ODEON diffuse model. Also shown in this figure is a horizontal line across the plot area that corresponds to an accuracy of 10%.

In the present study, the classroom used for model calibration was the simulated classroom fitted with the sound-absorbing configuration (50). In both programs, sound decays varied not only with the diffusion coefficient but also according to the surfaces that are assigned as diffuse reflectors. A first attempt in the model calibration was to consider all the room surfaces as diffuse reflectors. In ODEON, however, a slightly smaller average relative error in reverberation time prediction was obtained by modeling only the ceiling and the front wall as such in all configurations. These same surfaces were then modeled as diffuse reflectors also in RAYNOISE.

Unlike ODEON, and in accord with physical reality, RAYNOISE treats the diffusion coefficient as a frequency-dependent quantity. This gives greater flexibility for model calibration with RAYNOISE. This characteristic allowed the average relative error in reverberation time prediction across the six octave frequency bands from 125 Hz to 4 kHz to be reduced to 1.8% in the room chosen for calibration. In ODEON, because the diffusion coefficient is set as a frequency-independent quantity, the average relative error could only be reduced to 17.1% in the same room.

Figure 5 shows the average relative error in reverberation time prediction for RAYNOISE and ODEON, across the six octave bands from 125 Hz to 4 kHz, using both specular and calibrated diffuse models in all configurations. This figure reveals that with both programs, neither the specular model nor the calibrated diffuse model could predict reverberation times in the simulated classroom with an accuracy of 10%, in any of the sound-absorbing configurations. The only exceptions are the ODEON diffuse model prediction for configuration (0), and the RAYNOISE diffuse model prediction for configuration (50). The latter is an expected and obvious result since this configuration was used to calibrate the diffuse model.

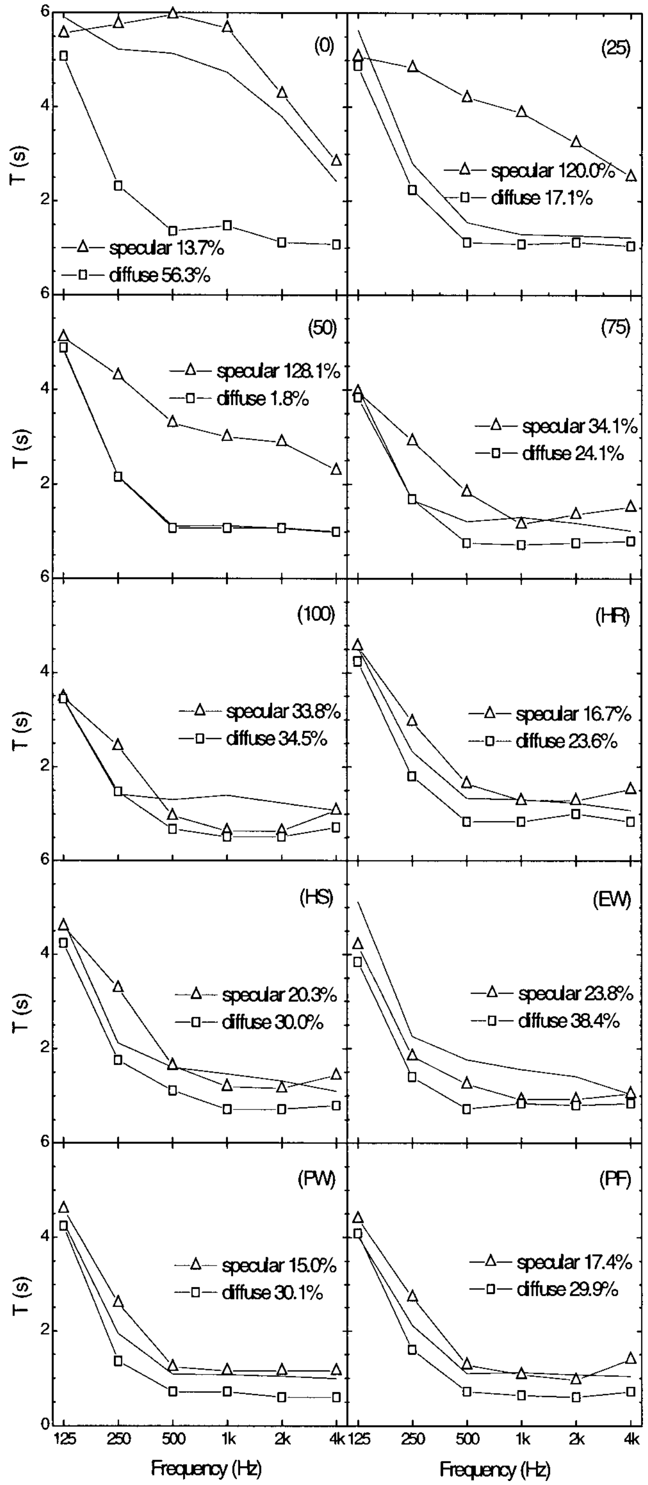

FIG. 6. (———) Experimental and RAYNOISE predictions of reverberation time. (△) RAYNOISE specular model; (□) RAYNOISE diffuse model. Indicated in each configuration is the average relative error of the prediction across the six octave bands from 125 Hz to 4 kHz.

Figures 6 and 7 compare experimentally obtained reverberation times with those predicted by RAYNOISE and ODEON, respectively, using both specular and calibrated diffuse models. Figure 6 shows that, for some sound-absorbing configurations, the RAYNOISE specular model is a more accurate predictor of reverberation time than the diffuse model, while for others, the calibrated diffuse model predictions are more accurate. Figure 7, which shows ODEON predictions, reveals that the calibrated diffuse model is more accurate than the

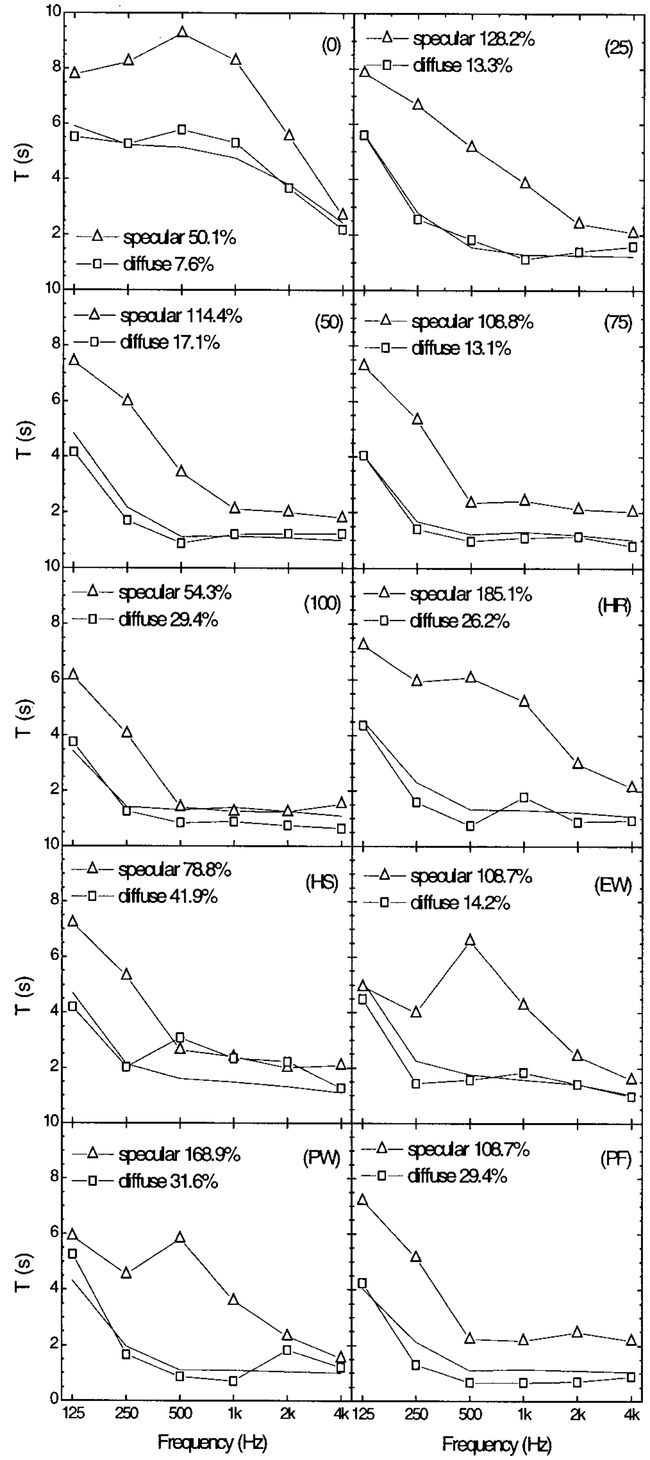

FIG. 7. (———) Experimental and ODEON predictions of reverberation time. (△) ODEON specular model; (□) ODEON diffuse model. Indicated in each configuration is the average relative error of the prediction across the six octave bands from 125 Hz to 4 kHz.

specular model in all configurations. Figure 7 also shows that the ODEON specular model consistently overpredicts reverberation times in the simulated classroom in all configurations.

Table II shows the overall average relative errors in reverberation time predictions for each computer program and reflection procedure. These average values do not include the room chosen for calibration—configuration (50). Similar to Table I, which gives the average relative errors of the different absorption exponents, Table II also shows that as more bands are included in the averages, the relative errors of the computer models are also reduced.

TABLE II. Overall average relative errors of the computer predictions of reverberation time for ten sound-absorbing configurations in the simulated classroom.

		Overall average relative error (%)		
		Frequency bands included in the averages		
Computer program		1 kHz	500 Hz–2 kHz	125 Hz–4 kHz
RAYNOISE 3.0	Specular	40.1	37.8	32.8
	Diffuse	44.7	42.7	31.6
ODEON 2.6	Specular	136.7	133.5	110.2
	Diffuse	29.5	30.8	23.0

The data in Table II do not provide a basis to rank the relative adequacy of the specular or the diffuse calculation procedures with RAYNOISE. This is because the RAYNOISE specular and calibrated diffuse models predict reverberation times with practically equal average relative errors, with values of 32.8% and 31.6%, respectively. The ODEON calibrated diffuse model gave the smallest average relative error of the computer predictions, with a value of 23.0%. On the other hand, the ODEON specular model gave the largest average relative error of the computer predictions, with a value of 110.2%.

Dance and Shield[12] also found that the average relative errors in reverberation time prediction of three computer programs in a concert hall were 14.0%, 14.3%, and 18.2%, using the standard absorption coefficients in the simulations. When the Millington absorption coefficients were used in the computer simulations, these errors were reduced, respectively, to 11.0%, 7.2%, and 11.7%. These errors are smaller than the smallest average relative error of the computer simulations obtained by the present study with the ODEON calibrated diffuse model (23.0% average relative error).

In an international round robin of 14 ray-based computer programs,[21] most programs predicted reverberation times in the 1-kHz octave band with relative errors between 5% and 10%, when absorption and diffusion coefficients were given. However, a considerably larger scatter of the results was obtained when the user had free choice of absorption coefficients.

D. Effect of sound-absorption treatments

Table III shows the average reverberation time at mid-frequencies for the ten sound-absorbing configurations in the simulated classroom. It can be seen that when sound absorption is added to the bare room [configuration (0)], a significant reduction in the average reverberation time is achieved, from 4.4 to 1.4 s, by covering 25% of the floor area with ceiling tiles [configuration (25)]. As more absorption is added, the corresponding reductions in the average reverberation times are not as significant. For instance, increasing absorption by a factor of 4 [from (25) to (100)] does not lower the average reverberation time by the same factor. In

TABLE III. Average reverberation times at midfrequencies (500 Hz–2 kHz) measured in the simulated classroom with different sound-absorbing configurations.

Sound-absorbing configuration	Average reverberation time at midfrequencies (500 Hz–1 kHz) (s)
(0)	4.4
(25)	1.4
(50)	1.1
(75)	1.2
(100)	1.3
(HR)	1.3
(HS)	1.4
(EW)	1.6
(PW)	1.1
(PF)	1.1

the simulated classroom, the corresponding reduction in the average reverberation time was from 1.4 to 1.3 s.

Also according to Table III, the shortest average reverberation time was not achieved when the room was most absorbing [configuration (100)]. The shortest average reverberation time, with a value of 1.1 s, was achieved when 50% of the floor area was covered [configuration (50)]. An average reverberation time value of 1.1 s was also achieved when the same amount of absorption that corresponds to configuration (50) was applied in other configurations [configurations (PW) and (PF)]. It is surprising that when this same amount of absorption was used in configurations (HR), (HS), and (EW), the average reverberation times were amongst the longest measured in the simulated classroom, with values of 1.3, 1.4, and 1.6 s, respectively. These results reveal that concentration of sound absorption on some room surfaces, as occurs in configurations (HR), (HS), and (EW), tends to produce longer reverberation times than those obtained when the same amount of absorption is more uniformly distributed, as in configurations (50), (PW), and (PF).

As a summary, the reverberation times that were measured in the simulated classroom for different sound-absorbing treatments show that increasing the amount of absorption does not necessarily produce reductions in reverberation time as predicted by the Sabine formula. This formula also predicts the same reverberation time for the same total absorption. Nevertheless, in some cases, different reverberation times were measured when the same amount of absorption was used in different configurations.

In a previous study,[8] a reverberation time of around 0.5 s was recommended for 100% speech intelligibility in very quiet classrooms. None of the absorptive treatments that was tested in the simulated classroom could produce the recommended reverberation time. As discussed above, the shortest average reverberation time at midfrequencies measured in the simulated classroom was 1.1 s, and it was not for the most absorbing configuration [configuration (100)]. This average reverberation time was measured by covering an area equal to 50% of the ceiling area in three different configurations [configurations (50), (PW), and (PF)].

V. SUMMARY AND CONCLUSIONS

The present study has evaluated the ability of several analytical expressions and two room acoustics computer programs to predict reverberation times measured in a simulated classroom. Although many reverberation time prediction schemes assume diffuse sound fields, these rarely occur and it is therefore important to evaluate predictions of reverberation times for more realistic conditions.

Increased sound scattering in the simulated classroom led to reduced reverberation times and better agreement with the predictions of the Sabine/Eyring formula. Increased scattering surfaces led to more diffuse conditions that better approximated the case of ideal diffusion on which these analytical expressions are based. However, even when the scattering frequency was increased up to 0.042 m^{-1}, complete agreement between measured and predicted values could not be achieved. It is therefore important that the conditions in the simulated classroom approximated those in a real unoccupied classroom. In these tests, conditions corresponded to a scattering frequency of 0.014 m^{-1} and this was assumed to represent conditions in a typical unoccupied classroom.

In the current evaluations of reverberation time predictions, it was assumed that a prediction accuracy of 10% would be satisfactory for most practical situations. However, none of the analytical expressions or the computer models could consistently predict reverberation times within this accuracy. Because these results were obtained for measurements in a simulated unoccupied classroom, they should not be generalized to other conditions.

The range of the average relative errors of the most accurate absorption exponents was found to lie approximately between 17% and 25%. The average relative error of the Sabine formula was 21.5%. In the simulated classroom, the Sabine formula consistently underpredicted reverberation times for all of the tested sound-absorbing configurations. The inaccuracy of the Sabine formula in the present case seems to be due to the sound field being less than ideally diffuse. Sound scattering was found to be a factor, but other factors including the amount and distribution of sound-absorbing material also appear to play a major role in the degree of diffusion of the sound field. There seems to be an interaction of these factors that results in unreliable predictions of reverberation times using the Sabine formula and other formulas based on the diffuse-field assumption.

Better reverberation time predictions were those that indicated longer reverberation times than the Sabine formula. However, none of the absorption exponents was within the required 10% accuracy and none could be singled out as more accurate for the ten different absorption configurations tested. They had similar average relative errors.

The RAYNOISE specular and calibrated diffuse models predicted reverberation times with similar average relative errors of 32.8% and 31.6%, respectively. Neither was more accurate than the other. Reverberation time predictions with the ODEON specular model were particularly inaccurate and the 110.2% average relative error of this method was the highest of all the predictive methods. However, the ODEON calibrated diffuse model consistently gave more accurate predictions than the ODEON specular model and had the smallest average relative error of all of the computer model predictions. The average relative error of the ODEON calibrated diffuse model was 23.0%, which is comparable with the accuracy of the best analytical expressions.

Covering the total floor area of the simulated classroom with highly absorbing ceiling tiles was not sufficient to achieve the recommended reverberation time for classrooms of 0.5 s.[8] For this condition, the measured midfrequency reverberation time was 1.3 s. Three different configurations that covered the equivalent of half of the ceiling area reduced the midfrequency reverberation time to 1.1 s. Various special configurations of the absorbing material that have been recommended in previous studies were not found to be significant improvements.

ACKNOWLEDGMENTS

The first author is spending sabbatical leave, granted by the University of São Paulo, São Paulo, Brazil, with the Acoustic Group at the National Research Council—NRC Canada. The first author would like to acknowledge the personal financial support for the development of the classroom acoustic project currently underway, granted by Fundação de Amparo à Pesquisa do Estado de São Paulo—FAPESP. The generous donation of the sound-absorbing material by Ottawa Fibre is gratefully acknowledged, as is the assistance of Keith Lay in making the reverberation measurements.

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