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A SIMPLE FORMULA FOR PREDICTING THE REVERBERATION TIME IN LONG ENCLOSURES

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Abstract

Reverberation time is one of the key elements for assessing the perception of sound in an enclosed space. It affects the noise propagation, speech intelligibility, clarity index, and definition. Since sound field in the long enclosure is non diffuse, classical room acoustics theory does not apply in long enclosures. Since 1960s, a number of theoretical investigations relating to sound propagation in long enclosures have been carried out but most of theses formulations are time consuming and too complex for routine use at a preliminary design stage. In order to simplify the numerical analysis, a simple yet sufficiently accurate numerical formula is developed that can be used to predict the reverberation time with a simplified formulation. Field measurements have been conducted in a model tunnel and a long corridor to explore the validity of the proposed numerical scheme.

1. INTRODUCTION

Reverberation time is often used for determining the quality of sound perception in an enclosure because it gives an estimation of the level of sound absorption that affects the speech intelligibility, clarity index and definition. Houtgast and Steeneken[1, 2] developed the concept of modulation transfer function, which was based on the reverberation time and background noise level. It was used to assess the quality of speech transmission in a communications channel. Their method established a physical parameter known as the speech transmission index that is commonly used today for rating the intelligibility of a sound source. It is worth noting that the classical Sabine theory[3] is used for more than a century to predict the reverberation time. Based on the Sabine theory, other studies were carried out to improve its accuracy in different room environments.[4-7] More recently, the ray tracing technique and the image source method are two popular approaches that have been used to develop numerical models for predicting reverberation times in rooms.[8-10] Despite these efforts, the Sabine formula is still used today, especially, in the preliminary design stage because of its simplicity and ease of application. It can give a reasonable estimate of the reverberation time in rooms.

Recent studies[11,12] demonstrated that the Sabine theory was not able to give a reasonable estimate of the reverberation time for the situation involving a long enclosure. The current study is motivated by the need to improve the acoustic environments of corridors and underground train stations. Quite often, the information of the reverberation time and the clarity of speech in a long enclosure are required at the preliminary design stage. We endeavor to develop a simplified scheme for calculating the reverberation time in long enclosures which are lined with sound absorption materials.

2. THEORETICAL FORMULATIONS

By using an image source approach, Kang[13,14] developed a numerical model to predict the reverberation time in a long enclosure. An equivalent continuous sound level of an impulse response at the receiver can be obtained by summing contributions of all image sources. Kang further simplified the process by applying a statistical method for estimating contributions from the image sources. In order to simplify the calculation, the statistical method has also been developed by Kang. The idea of the statistical method is as follows. First, the average distances from the image sources to receiver are determined. Secondly, the number of image sources that contributes in each time step is estimated. Finally, the average number of reflections for the image sources located in each step is approximated. Together with the geometrical and acoustical information of the long enclosures, it is then possible to estimate the reverberation time for a given source/receiver configuration.

In our current study, we follow the idea of the statistical method to derive a simplified formula to approximate the reverberation time in long enclosures. Suppose that the source and receiver are located, respectively at (0, 0, 0) and (x_r, y_r, z_r), i.e., their horizontal separation is z_r. The long enclosure has a rectangular cross-sectional area with the origin located at the bottom left corner of the long enclosure in the source plane. The long enclosure has a width w and a height h, see Figure 1 for the geometrical configuration of the problem.

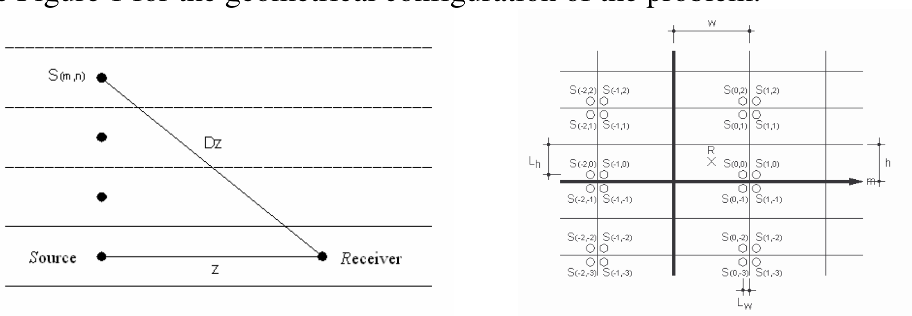


Figure 1: Image sources and receiver in an infinite long enclosure.

Infinite columns of image sources (four columns are shown in Figure 1) are formed because of the reflections from the two vertical walls. Each of these columns consists of a series of image sources because of the reflections from the two horizontal walls. We can link all image sources to the receiver establishing the respective ray paths. By the principle of reciprocity, it is possible to image that crests of spherical waves emanate from the receiver that arrive at different image sources at different times.

When the horizontal separation between the source and receiver become large, we can replace all image sources by an equivalent area source. For instance, if the omni-directional has unit strength then the area source has an equivalent strength of $1/wh$ per unit area where w is the width (measured in the x-axis) and h is the height (measured in the y-axis) of the cross-sectional area as shown in Figure 1.

The prime objective of the current study is to estimate the reverberation time for a given geometrical configuration of a rectangular enclosure. It is sufficient just to consider that both the source and receiver are located at their geometric centres of the respective planes. Without loss of generality, we take the initial time (t = 0) to be the moment when the wave crest emanating from the receiver, R, reaches the source plane at the point S. Figure 2 shows the position of the wave crests at an arbitrary time, t which is located at a radius of $r(t)$ from the point S. By a simple geometrical consideration, we can obtain the following relationship:

$$D(t) = D_0 + ct = \sqrt{r^2 + D_0^2} \quad , \tag{1}$$

where $D(t)$ is the path length from the image source to the receiver and D_0 is the corresponding path length at $t = 0$.

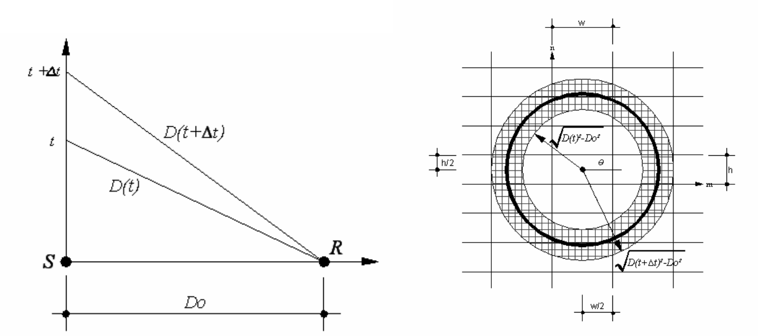


Figure 2: Calculation of r and $D(t)$ between time t and $t + \Delta t$.

For a general situation where a small area of image sources (of size δx by δy) is located at $(x, y, 0)$, the mean squared pressure, p^2, can be estimated

$$p^2 = P_0^2 \frac{D_0^2 \left(1-\bar{\alpha}\right)^{m+n}}{D^2} \frac{\delta x \delta y}{hw} \quad , \tag{2}$$

where P_0^2 is the mean squared pressure of the direct wave from the source to receiver and $\bar{\alpha}$ is the mean absorption coefficients of the walls. The parameters, m and n, are the number of reflections of the sound rays on the vertical and horizontal walls respectively. They can be approximated by

$$m = |x|/w \qquad \text{and} \qquad n = |y|/h . \tag{3a,b}$$

For the determination of the reverberation time, the source is turned on for a finite duration, Δt. Hence, for any time t, only a finite size of an area source contributes to the total sound field. The size of the area is dependent on Δt when the source is turned on. It is found more convenient to use a polar co-ordinate centering at the source point S. With the use of Eq. (2), p^2 at time t can be expressed by summing the sound energy incoherently as follows:

$$p^2(t) = \frac{2P_0^2 D_0^2}{hw} \int_{r(t)}^{r(t+\Delta t)} \int_{-\pi/2}^{\pi/2} \frac{(1-\bar{\alpha})^{r(\cos\theta/w + \sin\theta/h)}}{D^2} r dr d\theta . \tag{4}$$

In many cases, the long enclosure has relative hard boundary surfaces, i.e. $\bar{\alpha}_v \to 0$ and $\bar{\alpha}_h \to 0$. We can treat r, which appears in the reflection term $\left(1-\bar{\alpha}\right)^{r(\cos\theta/w+\sin\theta/h)}$ of Eq. (4), as constant. Then, the integral with respect to r can be evaluated to give

$$p^2(t) = \frac{\pi P_0^2 D_0^2}{hw} \ln[D^2(t+\Delta t)/D^2(t)] \bar{V} , \tag{5}$$

where $\bar{V}$ is the term representing the mean reflection factor. It is given by

$$\bar{V} = \int_{-\pi/2}^{\pi/2} \left[\left(1-\bar{\alpha}\right)^{m+n} \right]^{\cos\theta} d\theta \Big/ \pi \tag{6}$$

where $m\left(\equiv r(t)/w\right)$ and $n\left(\equiv r(t)/h\right)$ can be regarded, respectively, as the maximum number of possible reflections at the horizontal and vertical walls. The following identity has been used to derive Eq. (6):

$$\int_0^{\pi/2} \left(1-\bar{\alpha}\right)^{n\cos\theta/h} d\theta = \int_0^{\pi/2} \left(1-\bar{\alpha}\right)^{n\sin\theta/h} d\theta \quad . \tag{7}$$

Generally, there is no exact analytical solution for the reflection factor given in the integral of Eq. (6). Kang suggested that the reflection factor should be approximated by

$$\bar{V} = \left(1-\bar{\alpha}\right)^{\bar{m}+\bar{n}} \tag{8}$$

where $\bar{m}+\bar{n}$ is the average number of reflections for those image sources that have contributed to the total sound field. In the context of an area source, we can show that

$$\bar{m} = 2m/\pi \qquad \text{and} \qquad \bar{n} = 2n/\pi \, . \tag{9a,b}$$

Noting the relationship $r(t) = wm = hn$ and the above equations, we can show that

$$\bar{m} = 2r(t)/w\pi \qquad \text{and} \qquad \bar{n} = 2r(t)/h\pi \, . \tag{10a,b}$$

To allow the computation of sound fields given in Eq. (5), the time step Δt of the impulse sound is required. The parameter Δt should be sufficiently short for the determination of the reverberation time in long enclosures. We find it useful to define Δt as the time step required for the wave crest to travel one image source distance, and the Δt can be assumed as,

$$c\Delta t = \Delta r = \sqrt{wh/\pi} \tag{11}$$

Then, $D(t)$ and $D(t+\Delta t)$ in Eq. (5) are given as follows:

$$D^2(t) = D_0^2 + r(t)^2 \qquad \text{and} \qquad D^2(t+\Delta t) \approx D_0^2 + r^2(t) + 2r(t)\Delta r \tag{12a,b}$$

where the assumption of $r(t) >> \Delta r$ has been used in Eq. (12b). The mean squared pressure can then be simplified by substituting Eqs. (8) – (12) into Eq. (5) to give

$$p^2 = \frac{\pi P_0^2 D_0^2}{wh} \ln\left[1 + \frac{2r\sqrt{wh/\pi}}{D_0^2 + r^2}\right](1-\bar{\alpha})^{(2r/\pi)(1/w+1/h)} . \tag{13}$$

When the image sources are located further away from the center, i.e. $r >> wh$, Eq. (13) can then be rewritten in terms of relative sound pressure level, L_r as

$$L_r = 10\log_{10}\left\{\frac{\pi D_0^2}{wh} \ln\left[1 + \frac{2r\sqrt{wh/\pi}}{D_0^2 + r^2}\right]\right\} + (2r/\pi)(1/w+1/h)\log_{10}(1-\bar{\alpha}) , \tag{14}$$

where the relative sound pressure level is defined as the ratio of p to P. It is given by

$$L_r = 20\log_{10}\left[p/P_0\right] \tag{15}$$

The reverberation time can be determined for the time when L_r is reduced by 60 dB. The path length at time $t = T_{60}$ can be determined from Eq. (1) to yield

$$D_T = \sqrt{{r_T}^2 + {D_0}^2} \tag{16}$$

where the subscript T denotes the corresponding parameters at T_{60}. Making using of Eq. (14) and noting L_r = - 60 dB, r_T can be found by solving the equation as follows

$$f(r_T) = 0 , \tag{17a}$$

where
$$f(r_T) = 10^6 \times \frac{\pi D_0^2}{wh} \ln\left[1 + \frac{2r_T\sqrt{wh/\pi}}{D_0^2 + r_T^2}\right](1-\bar{\alpha})^{2(r_T/\pi)(1/w+1/h)} - 1 . \tag{17b}$$

Substituting r_T it into Eq. (16), we can determine the reverberation time, T_{60}, as follows

$$T_{60} = (\sqrt{{r_T}^2 + {D_0}^2} - D_0)/c \quad . \tag{18}$$

For absorptive boundary surfaces of a long enclosure, we have $\bar{\alpha} \to 1$ and $r_T \to 0$ according to Eq. (17b). Thus, from Eq. (18), the reverberation time, T_{60}, becomes zero which is consistent with the expectation. On the other hand, if the mean absorption coefficient is small for reflective boundary surfaces, then $r_T >> D_0$ in this case. We can approximate Eq. (17b) by

$$f(r_T) = 10^6 \times \frac{2D_0^{\ 2}\sqrt{\pi}}{r_T\sqrt{wh}}(1-\overline{\alpha})^{2(r_T/\pi)(1/w+1/h)} - 1 \quad . \tag{19}$$

For a rigid surface, i.e. $\alpha = 0$, we can solve r_T directly from Eq. (17a) to give

$$r_T = 2\times 10^6 \times D_0^{\ 2}\sqrt{\pi / wh} \quad . \tag{20}$$

Hence, the reverberation time, T_{60}, is given by

$$T_{60} = r_T / c = 1.04\times 10^4 \times D_0^{\ 2} / \sqrt{wh} \quad , \tag{21}$$

where the speed of sound in air is taken as 340 m s^{-1}. We see that T_{60} is proportional to the square of the horizontal separation between the source and receiver and inversely proportional to the cross-sectional area of the long enclosure. For non-rigid boundary surfaces, Eq. (19) can be solved straightforwardly for r_T in which the reverberation time can be determined.

3. FIELD MEASUREMENT

In order to validate the proposed methods, field measurements were conducted in a long corridor and in a model tunnel of size of 28.5 m long, 1.16 m width and 1.46 m height. Due to a relatively high background noise levels (> 40 dB), T_{30} is used to obtain the reverberation time in the present study. In other words, instead of determining the required time for the reduction of noise levels by 60 dB, we measured the decay time for the reduction of noise levels by 30 dB only. We then extrapolate to obtain the parameter for T_{60}. To assess the contribution from the direct fields, we also measure the background noise level is usually very high in long enclosures.

3.1 Field measurements – a long corridor

Measurements of the reverberation time were conducted in a corridor in the Department of Mechanical Engineering, the Hong Kong Polytechnic University. The corridor is 35.6 m long, 1.53 m width and 2.45 m height. The floor was covered with carpet and the ceiling was made up of perforated panels filled with fibreglass. The vertical partition walls were finished with plaster, and wooden doors. Published data for the absorption coefficients of different materials were used to estimate the average absorption coefficient of the boundary walls. These published data were obtained from a standard acoustic handbook. The averaged absorption coefficients from 200 Hz to 4000 Hz are listed in Table 1. All data are given in one-third octave bands.

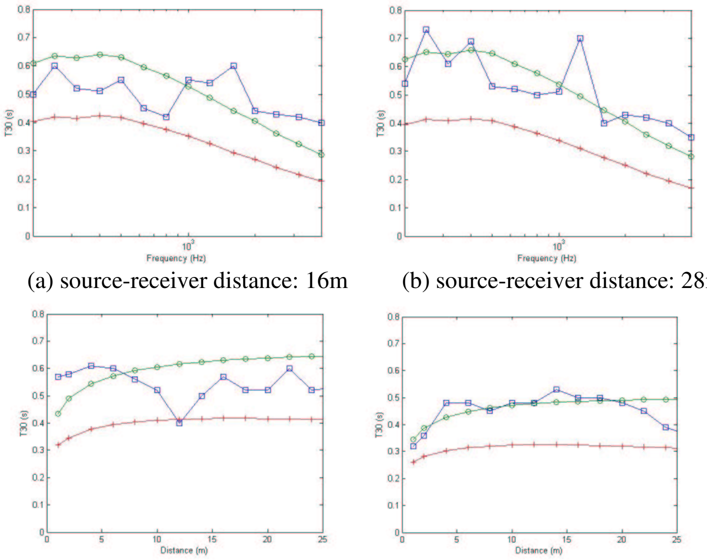
Frequency (Hz)	**200**	**250**	**315**	**400**	**500**	**630**	**800**
Mean absorption coefficient	0.0922	0.0887	0.0895	0.0879	0.0894	0.0942	0.099
Frequency (Hz)	**1000**	**1250**	**1600**	**2000**	**2500**	**3150**	**4000**
Mean absorption coefficient	0.1054	0.1135	0.1244	0.1344	0.1491	0.164	0.1814

Table 1: Mean absorption coefficient of the corridor.

A Brüel & Kjær type 4942 pre-polarized diffuse field 1/2-inch condenser microphone was used as the receiver and a Tannoy T300 loudspeaker was used as the point source in this measurement. A PC-based maximum length sequence system analyzer (MLSSA)[15] was used both as the signal generator for the source and the analyzer for subsequent data processing. Its post-processing functions calculate most of the acoustical parameters such as reverberation time and speech transmission index from the measured impulse response. The loudspeaker was located 4 m from one end of the corridor, along the centreline of sidewall and 0.95 m above floor. The receiver was placed along the centreline at 2 m intervals, and 1.2 m height.

In the plots presented below, we shall show comparisons of experimental data with numerical predictions based on two comparable numerical schemes. The first scheme is discussed in Section 2 above and it is referred as the integration formula. The second numerical scheme is based on Kang's model [13] and we refer it as the summation model.

Figures 3 (a) – (d) show comparisons of experimental results with the predicted reverberation time in long enclosures. Figure 3(a) and 3(b) show the predicted T_{30} spectra. The separation between the source and receiver is 16 m for Figure 3a and 28m for Figure 3b. Figures 3c and 3d display the predicted T_{30} versus the horizontal separation with the source frequency of 500 Hz and 2 kHz respectively. As shown in these two figures, the numerical predictions according to the integration formula and the summation model are presented. In general, better agreements are obtained for the experimental data with the integration formula than the summation model. For the reverberation time spectra [Figures 3(a) and 3(b)], the integration formula over-predicts T_{30} of an average of 0.9 s for the source/receiver separation of 16m and 0.7 s for the separation of 28 m. On the other hand, the summation model according Kang under estimates the respective T_{30} of 0.16 s and 0.2 s respectively.



(a) source-receiver distance: 16m (b) source-receiver distance: 28m

(c) frequency: 500 Hz (d) frequency: 2 kHz

Figure 3: Comparison of measured and predicted T_{30} in the corridor.
(squares: measurements; circles: proposed methods; plus signs: Kang' image source method [13])

In Figure 3(c), the average 'errors' of the current method (the integration formula) and Kang's approach (the summation model) are quite similar. However, the integration model over estimates the T_{30} of about 0.1 s but the summation model under estimates the reverberation time by about 0.15 s. Finally, Figure 3(d) exhibits the a very good agreement between the integration formula with the measured data. For this case, the agreement between the summation model and the experimental is less impressive. However, the summation model only under estimates the reverberation by 0.18 s.

With these typical experimental data, we can see that the proposed method, in general, agrees better with experimental measurements than those predicted by the summation method.

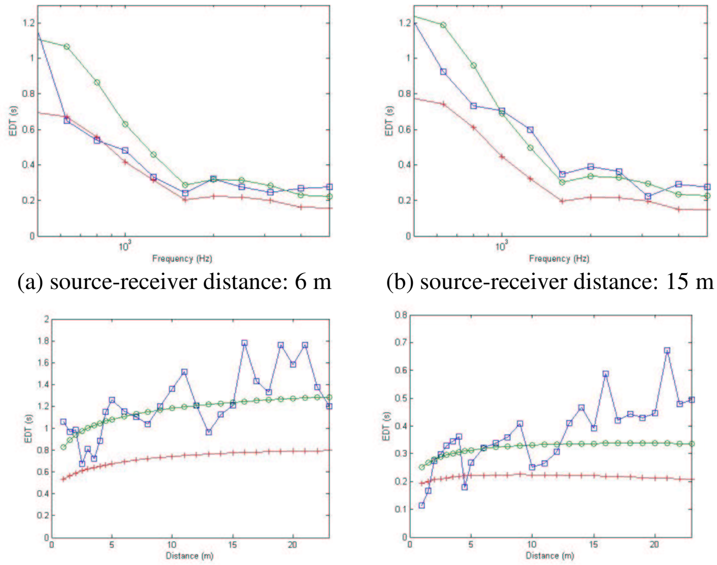
B. Indoor measurements – a model tunnel

A 28.5 m long model tunnel with a cross-sectional area of 1.16 m (width) by 1.46 m (height) was constructed for verification of the integration formula developed in Section 2. The tunnel walls, floor and ceiling were fully covered by gypsum board. The same set of equipment was also used in the experiments conducted in the model tunnel. Due to the size of the model tunnel, a Renkus-Heinz PN81 self-powered loudspeaker was used in the experimental measurements. The speaker was smaller in size that was suitable for this model tunnel. Prior measurements were conducted to determine the absorption coefficient of gypsum board after the field measurement. The measured absorption coefficients of gypsum board from 125Hz to 6300Hz are listed in table 2.

Frequency (Hz)	200	250	315	400	500	630	800
Mean absorption coefficient	0.0019	0.0227	0.0254	0.0249	0.0324	0.0338	0.0419
Frequency (Hz)	**1000**	**1250**	**1600**	**2000**	**2500**	**3150**	**4000**
Mean absorption coefficient	0.0581	0.08	0.126	0.114	0.116	0.128	0.156

Table 2: Mean absorption coefficient of gypsum board in model tunnel.

The noise source was placed off-set from the centre line at a distance 0.86 m from one of the vertical walls. It was placed at a height of 0.4 m above the ground. The receiver was placed at centreline of the tunnel at a height of 0.4 m above ground and at an interval of 1 m from the source. Due to the possible resonance effect of the model tunnel, measured data below 500 Hz were not presented.



(a) source-receiver distance: 6 m (b) source-receiver distance: 15 m

(c) frequency: 500 Hz (d) frequency: 2 kHz

Figure 4: Comparison of measured and predicted EDT in the model tunnel.
(squares: measurements; circles: proposed methods; plus signs: Kang' image source method)

In this set of indoor measurements, we only present the experimental results for the Early Decay Time (EDT) and their comparisons with the two numerical models: the current model (integration formula) and Kang's model (Summation model). Figure 4(a) and 4(b) show the measured EDT spectra at a horizontal separation of 6 m and 15 m respectively. At the horizontal separation of 6 m, the predicted EDT according to the two numerical models agrees very well with each other up to the frequency of 1600 Hz. Nevertheless, the agreements are reasonable well for all other frequencies. We note that the predicted EDT generally under-estimates the measured data with an average of 0.1 s. In Figure 4(b), we also see that the integration formula generally gives a better agreement with measured data than those predicted by the summation model.

In Figure (4c) and (4d), we show the EDT versus the horizontal distance between the source and receiver for a frequency of 500 Hz and 2 kHz respectively. Again, the integration formula shows a better agreement with the experimental data as compared with the summation model.

4. CONCLUSION

In the present, a simple analytic formulation has been derived to predict the reverberation time in a long enclosure. The formulation is developed to facilitate a preliminary stage for the acoustical design of a long enclosure. The proposed theory has been validated by comparing with the other comparable numerical formulations. The predictions also compare with experimental data in a corridor and in a model tunnel. It has been demonstrated that the proposed theory agrees tolerably well with experimental results and provides a more accurate predictions than those numerical model developed earlier.

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REFERENCES

[1] T. Houtgast and H. J. M. Steeneken, "The modulation transfer function in room acoustics as predictor of speech intelligibility," *Acustica* **28**, 66-73 (1973).
[2] H. J. M. Steeneken and T. Houtgast, "A physical method for measuring speech-transmission quality," *J. Acoust. Soc. Am.* **67**(1), 318-326 (1980).
[3] W. C. Sabine, *Collected papers on Acoustics* (Harvard University Press, Cambridge, MA, 1927).
[4] C. F. Eyring, "Reverberation time in dead rooms," *J. Acoust. Soc. Am.* **1**, 217-241 (1930).
[5] G. Millington, "A modified formula for reverberation," *J. Acoust. Soc. Am.* **4**, 69-82 (1932).
[6] D. Fitzroy, "Reverberation formula which seems to be more accurate with nonuniform distribution of absorption," *J. Acoust. Soc. Am.* **31**, 893-897 (1959).
[7] H. Arau-Puchades, "An improved reverberation formula," *Acustica* **65**, 163-180 (1988).
[8] S. M. Dance, J. P. Roberts, and B. M. Shield, "Computer prediction of sound distribution in enclosed spaces using an interference pressure model," *Applied Acoustics* **44**, 53-65 (1995).
[9] S. M. Dance, "The representation of fittings in the computer prediction of sound propagation in enclosed spaces," *Applied Acoustics* **52**(1), 71-83 (1997).
[10] S. M. Dance, "Minimal input models for sound level prediction in fitted enclosed spaces," *Applied Acoustics* **63**, 359-372 (2002).
[11] T. Yamamoto, "On the distribution of sound in a corridor," *J. Acoust. Soc. Japan.* **17**, 286-292 (1961).
[12] P. Steenackers, H. Myncke and A. Cops, "Reverberation in Town Streets," *Acustica* **40**, 115-119 (1978).
[13] J. Kang, "Reverberation in rectangular long enclosures with geometrically reflecting boundaries," *Acustica* **82**, 509-516 (1996).
[14] J. Kang, *Acoustics of long spaces* (Thomas Telford Publishing, London, 2002).
[15] D. D. Rife and J. Van der Kooy, "Transfer-function measurement with Maximum-Length sequences," *J. Audio Eng. Soc.* 37, 419-443 (1989).