Repeatability and Standard Error in Reverberation Chamber Absorption Coefficient

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Abstract

In order to check that the reverberation chamber measurement of absorption coefficient was consistent, a short run of repeatability measurements were carried out as per standard procedure. A sample of gypsum board panels was measured six times in the same configuration, being removed from the chamber and replaced for each measurement. The calculation of standard error was made to estimate uncertainty in the absorption coefficient. The standard error from calculation was a reasonable prediction of the uncertainty which was actually found if a given measurement was repeated several times.

1. Introduction

Yet the oldest parameter in room acoustics is still the one which hall designers turn to first. This is reverberation time which, as Sabine established empirically in 1923, should depend solely on the volume and total absorption of a room. Of all the parameters in common use in auditorium design today, reverberation time was the first to be established and it is one of the most subjectively important. Because the total absorption in a hall is dominated by that of the seating and audience, it is essential that these can be measured or predicted accurately in the early stages of design. However there is at present no wholly accepted calculations of repeatability and standard error in reverberation chamber absorption coefficient and the data quoted in the literature varies widely. The work reported here aims to clarify this situation.

2. The reverberation chamber measurement system

In validating a variation of a technological testing method, it is important that the accuracy of the equipment and facilities used should have a traceable standard. To this end, it was ensured that the reverberation chamber and equipment used for the laboratory measurements of sound absorption complied with IS 8225-1987 (ISO 354-1985) [1]. However, it is thought to be perfectly adequate in other aspects of equipment and method specification. The standard recommends a room volume of approximately 200 m^3. The room used at the Acoustics Section of NPL at New Delhi has a volume of 257 m^3, a surface area of 244 m^2 and the dimensions are roughly 6m x 6.5m x 7m which satisfy the requirements of ISO [1]. The room plan in Fig. 1 shows that it has no parallel walls, floor and ceiling to add diffusion; but the ceiling is slightly horizontal (i.e. the average inclination between walls being 6° and between floor and ceiling 2°-3°).

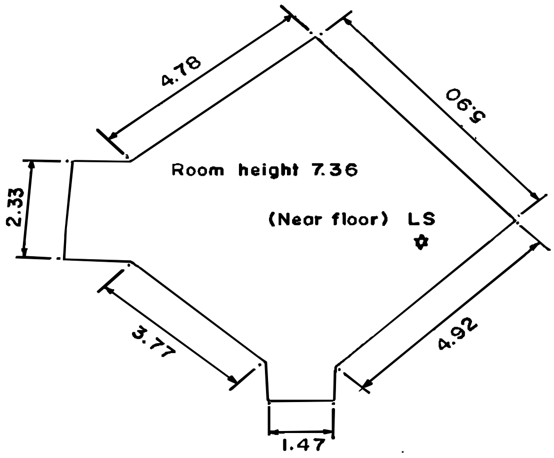

Figure 1: Floor plan of reverberation chamber.

In accordance with IS 8225, the aspect ratio of the room is not far from unity, and

$$l_{\max} < 1.9V^{1/3} \qquad (1)$$

where V is the volume of the room and l_{max} is the length of the longest straight line which fits inside it. Ten fixed diffusers were suspended in the room. These consisted of curved and varnished plywood sheets with a total projected area (i. e. equivalent to room floor area) of 35.1 m^3 oriented and hung at random throughout the room.

To measure sound decays in the room, two loudspeakers, indicated in Fig. 1, and five microphones, all at known positions were used. Broadband pink noise was radiated from one loudspeaker, interrupted, and the decay recorded at one microphone. This was done five times and the five decay curves were averaged in 1/3 octave bands by a Norwegian Electronics Real-time Analyser RTA-830. The analyser was then used to calculate the RT in each frequency band for this loudspeaker and microphone combination by fitting a straight line between the -5 and -35 dB points of the averaged decay curve. This procedure was repeated at the other four microphones. Next the microphones were moved to new known positions and the second loudspeaker was used to generate another five decays for each microphone. This process resulted in ten RTs in each frequency band which were used to give an ensemble average in each band. The measurement process thus involved averaging across time and space. Because all the evaluated RTs were recorded and kept, any systematic variations between different loudspeaker and microphone position combinations could be examined if poor homogeneity was suspected. It was assumed that any variations between the five decays at any one measuring position would be essentially random, and so these raw decays were averaged without being stored. Fig. 2 shows a typical RT spectrum for the empty room, along with the IS 8225 minimum curve for the particular room volume and surface area.

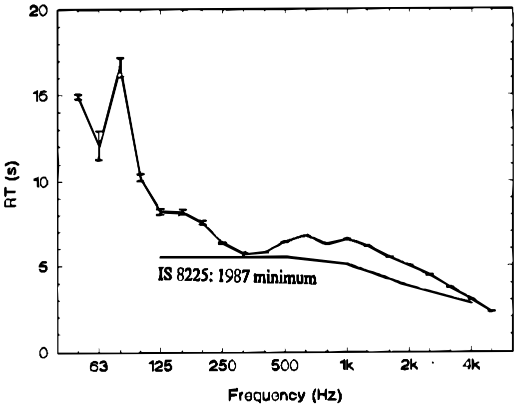

Figure 2: Averaged reverberation time in empty chamber plotted with IS 8225-1987 minimum curve. The error bars represent ± one standard error.

2.1. Reverberation time formula

The reverberation is formed by a multitude of reflected sounds superposed without discontinuity that add to the direct sound and prolong it. The reverberation time (T_r) is defined as time required for the sound energy density to decay 60 dB after the source has stopped emitting. It depends on the acoustic and geometric characteristics of the walls which bound the space. There are methods to determine reverberation time based on the wave or geometrical theory [2].

In diffuse sound field conditions, for a reverberant room with walls of a homogeneous geometrical and acoustic nature and for an omnidirectional source, Sabine [3] defines the reverberation time according to the average absorption coefficient of the walls $\bar{\alpha}$ as:

$$T_r = \frac{0.16 . V}{S\bar{\alpha}} \tag{2}$$

V and S are the volume of the chamber and the total wall area, respectively.

$\bar{\alpha}$ is the arithmetic average of the area elements S_i associated with the absorption coefficient α_i :

$$\bar{\alpha} = \frac{1}{S}\sum_i S_i \alpha_i \tag{3}$$

$\bar{\alpha}$ is the absorption coefficient normally used by professionals. It is usual to find $\bar{\alpha}$ higher than 1 for certain absorbent materials.

2.2. Absorption coefficient formula

Sabine's formula was used to calculate the absorption coefficient α of a sample from reverberation time measurements made with and without the samples present in the chamber:

$$\alpha = \frac{55.3V}{S(331+0.6t)}\left[\frac{1}{T_s}-\frac{1}{T_e}\right] \tag{4}$$

where S, t, T_s, and T_e are the plan area of sample in m^2; the average ambient air temperature in °C; the reverberation time measured with and without the sample in the chamber, in seconds respectively.

It should be remembered that this formula, though originally derived empirically by Sabine [3], is a product of statistical room acoustics which requires a perfectly diffuse sound field and ignores wave effects. These has been much criticism of its widespread use in room acoustics because of this for example [4] and many authors have proposed alternative formulae and methods. These range from the widely-accepted equation of Eyring [5], often used where average absorption is high, through less well-known and more complicated formulae like that of Arau-Puchades [6], to the complete refutation of theoretical statistical room acoustics by Gomperts. Certainly, it is true that perfect diffusion cannot be obtained with a highly-absorbing surface in the room, but the criticisms of Gomperts seem overstated. Because Sabine's formula remains dominant, at least for technological testing, it was felt important that any adaptation of the method for measuring seating absorption should also use it. It seemed that Ducourneau and Planeau's method [7] for measuring panel absorption might offer increased accuracy while still allowing a simple calculation formula. As with all statistical room acoustic methods, the main proviso is that errors in absorption coefficients estimated from the reverberation time variances may be smaller than the real uncertainties at low frequencies due to low modal density in the measuring room. Kuttruf [8] has contributed a formula for the lowest frequency at which the modal density in a room is high enough for statistical equations to be used with confidence:

$$\sqrt[3]{V}\,.\,f_{\min} \approx 1000 \qquad (5)$$

For the reverberation chamber used here, $f_{\min}$ is 157 Hz. It should be noted that reverberation rooms are often used below this limiting frequency. IS 8225 stipulates a lower 1/3 octave measurement band of 100 Hz, but recommends a room volume of 200 m^3, for which $f_{\min}$ is 171 Hz.

3. Repeatability of measured absorption coefficients

In order to check that the measurement system described above was consistent in itself, a short run of repeatability measurements was made, as recommended by IS 8225. A sample of 12.5 mm thick gypsum board panels (test sample area of 11.2 m^2) was measured six times in the same configuration, being removed from the chamber and replaced for each measurement. The repeatability "r" of the system was then found from

$$r = t\,\sqrt{2}.s_{n-1} \qquad (6)$$

where t is Student's factor for 95% probability and five degrees of freedom and σ_{n-1} is the standard deviation of the six measurements. The six absorption coefficient are plotted in Fig. 3, along with r and a typical standard error curve from one of the measurements. The average α value of repeatability (r_{95}) is 0.11 in terms of reverberation chamber absorption coefficient and 0.02 in terms of absolute error. The average α value of typical standard error is 0.02 in terms of absolute error as shown in Fig. 3.

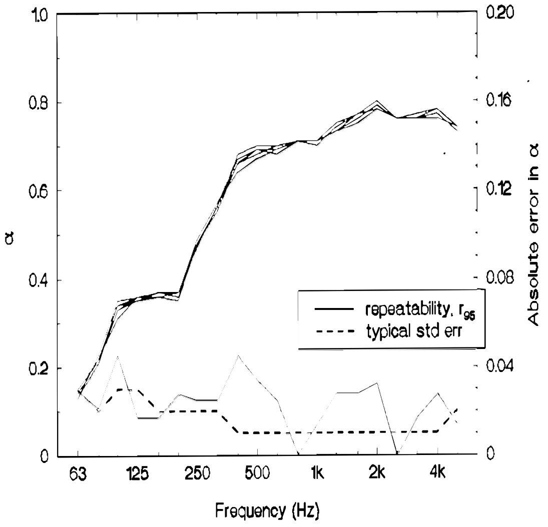

Figure 3: Repeatability (95%) of six gypsum tiles absorption measurements and typical standard error of one.

4. Standard error of absorption coefficient

The absorption coefficient calculated for the results is a compound quantity:

$$\alpha = \frac{0.161\,V}{S}\left[\frac{1}{T_f} - \frac{1}{T_e}\right] \qquad (7)$$

where the suffix f denotes that the sample is in the room and e that is empty. However, T_f and T_e are themselves compound quantities, derived from set of measurements made at five microphone positions for each of two source locations, A and B. Consider first the empty room, and drop the e suffix for clarity. For position A, measurements are made at five microphone positions, so the mean RT is

$$\overline{T_A} = \frac{1}{5}\sum_{i=1}^{5} T_{A_i} \qquad (8)$$

Now in general, for n observations, the standard error in the mean is [9]

$$s = \frac{\sigma_{n-1}}{\sqrt{n}} \qquad (9)$$

Hence, for the five measurements at position A,

$$s_A = \sqrt{\frac{\sum_{i=1}^{5} T_{A_i}^2 - 5(\overline{T_A})^2}{5 \times 4}} \qquad (10)$$

$$\Rightarrow \quad \sum_{i=1}^{5} T_{A_i}^2 = 20 s_A^2 + 5\overline{T_A}^2 \qquad (11)$$

and similarly, for position B,

$$\sum_{i=1}^{5} T_{B_i}^2 = 20 s_B^2 + 5\overline{T_B}^2 \qquad (12)$$

Now, T_{Ai} and T_{Bi} are equivalent (through their means and standard errors are not). So equations (11) and (12) may be added to give

$$\sum_{i=1}^{10} T_i^2 = 20(s_A^2 + s_B^2) + 5(\overline{T_A}^2 + \overline{T_B}^2) \qquad (13)$$

Now the empty room process can be considered as a whole to arrive at the empty room standard error. Reasserting the e suffix,

$$s_e = \sqrt{\frac{\sum_{i=1}^{10} T_{e_i}^2 - 10\overline{T_e}^2}{10 \times 9}} \qquad (14)$$

where

$$\overline{T_e} = \frac{\overline{T_A} + \overline{T_B}}{2} \qquad (15)$$

Substituting for the summation in (14) from (13),

$$s_e = \sqrt{\frac{20(s_{e_A}^2 + s_{e_B}^2) + 5(\overline{T_{e_A}}^2 + \overline{T_{e_B}}^2) - 10\overline{T_e}^2}{90}} \qquad (16)$$

This is the standard error in the empty room RTs. s_{eA} and s_{eB} are found from equation (10), T_{eA} and T_{eB} from equation (8), and T_e from equation (15). Exactly the same measurement procedure is followed with the sample in the room, so s_f is found from equation (16), with suffix f substituted for e.

Now in general, the standard error of any compound function $f(m_1, m_2, \dots, m_n)$ is s, where

$$s^2 = \left[\frac{\partial f}{\partial m_1}\right]^2 s_1^2 + \left[\frac{\partial f}{\partial m_2}\right]^2 s_2^2 + \dots + \left[\frac{\partial f}{\partial m_n}\right]^2 s_n^2 \qquad (17)$$

and so the total standard error in the absorption coefficient of equation (5) is finally given by

$$s_\alpha = \frac{0.161V}{S} \sqrt{\left[\frac{s_f}{\overline{T_f}^2}\right]^2 + \left[\frac{s_e}{\overline{T_e}^2}\right]^2} \qquad (18)$$

5. Results and discussion

The standard error curve was computed from the variance of the measured RTs according to the formulae derived. Because it is not significantly less than half the 95% r curve, the calculated standard error is a reasonable estimate of the uncertainty in the measurement. In other words, the standard error is a reasonable prediction of the uncertainty which is actually found if a given measurement is repeated several times.

6. Acknowledgements

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7. References

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