Buenos Aires – 5 to 9 September, 2016
Acoustics for the 21st Century…

PROCEEDINGS of the 22nd International Congress on Acoustics

Isotropy and Diffuseness in Room Acoustics: Paper ICA2016-556

A rigorous definition of the term “diffuse sound field” and a discussion of different reverberation formulae

Uwe M. Stephenson[a]

[a] HafenCity University, Germany, post@umstephenson.de

Abstract

Often, Sabine’s and other reverberation formulae are applied without really knowing whether the sound field is sufficiently diffuse. In this rather didactical paper a rigorous definition of the crucial term 'diffuse sound field' is proposed and the relationships to the necessary surface conditions, especially scattering, are analyzed. Also the reasons for the differences between Sabine's and Eyring's reverberation formulae are analyzed. Some other approaches for partially diffuse sound fields, e.g. Kuttruff’s formula, are discussed. Some numerical investigations are added. The aim is to find reliable definitions of conditions for at least an approximately diffuse sound field.

Keywords: room acoustics, reverberation theory, diffuse sound field

A rigorous definition of the term “diffuse sound field” and a discussion of different reverberation formulae

1. Introduction

The term "diffuse sound field" (DSF) is often not explained very accurately - the motivation for this paper. The paper provides the following sections: 2. a more rigorous definition is proposed and the relationships to the necessary surface conditions as absorption and scattering are discussed; 3. Some basic quantities are defined, especially the free path length - the derivation of which is systematized; 4. re-derivation of the Eyring and Sabine reverberation formulae; 5. analyzation of the reason for the difference between both formulae; 6. discussion of different transition models; 7. a short review of analytical and 8) semi-analytical reverberation models for only partially diffuse sound fields. The general condition of geometric/statistic room acoustics is that typical room dimensions are large compared with wavelengths such that the analysis may be performed with an energetic sound (particle) model (for one frequency band).

2. Conditions for the diffuse sound field

First, it should be distinguished between theoretical definitions and practical conditions, further between the claim the sound field should be diffuse 'from the start' (strict version) or 'towards the end of reverberation'' (tolerant version, conditions in brackets) (see Table).

Usually one starts with A (' each direction with same intensity’/‘directional diffusivity’). From A follows B (in a room without absorption, the particles don't lose energies, see the lines connecting the clusters in Fig. 1a [1], but not vice versa (consider e.g. the case of a long room homogeneously filled with rays but just in a longitudinal direction). The room does not need to be convex (the argumentation of Fig.1a could be extended by more array-clusters) - if a diffuse sound field is really given. However, in non-convex rooms with weakly coupled sub-spaces the sound field is hardly diffuse. From B (a volume condition) follows as a surface condition B2 (Fig. 1b).

The surface conditions C (zero absorption) +D (total scattering) are necessary but not sufficient for A+B- as all surfaces may be totally diffusely reflecting, but the irradiation strengths may be non-constant due to geometry (typically if the absorption distribution over the surface is quite uneven). If just one piece of surface is absorbing, then the sound field closely in front will not be isotropic. The same happens, strictly speaking, with only one specular reflecting peace of surface as producing a mirror image source and hence a singularity in the directivity.

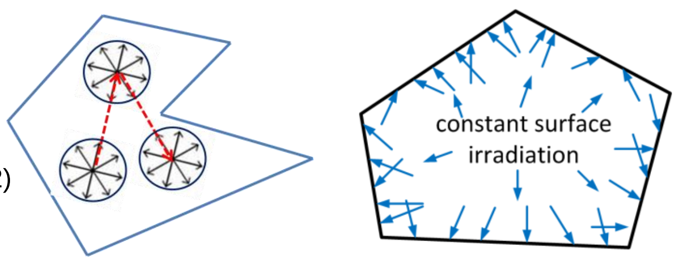
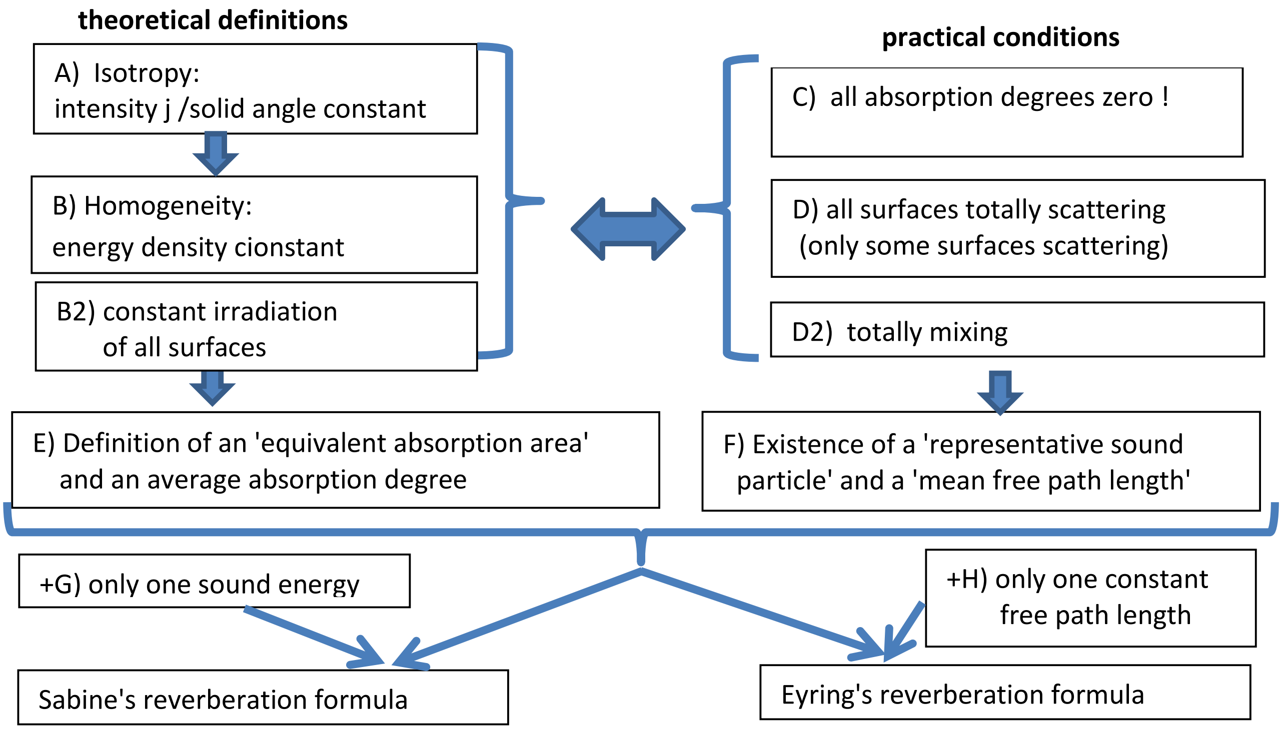


Fig. 1a: Isotropy and homogeneity
(same arrow lengths in every direction everywhere,at every 'array-cluster');
b (right): constant irradiation of all surfaces (B2) following from B) (homogeinity)

IBERO-AMERICAN FEDERATION of ACOUSTICS
INTERNATIONAL COMMISSION for ACOUSTICS
ARGENTINIAN ACOUSTICIANS ASSOCIATION

Table: Conditions for the diffuse sound field (in parentheses: tolerant version)



Total scattering means Lambert reflection: the reflection angle (ϑ) probability density p' per solid angle is proportional $\cos(\vartheta)$, independent from the incidence angle

$$p' =: \frac{dp}{d\Omega} = \frac{cos(\vartheta)}{\pi} \qquad \text{(π is the normalization factor for the half-sphere)} \qquad (1)$$

It can be considered as the ideal scattering characteristics of 'rough surfaces' following from the cos- projection law and the reciprocity principle. It can be shown that just this cos-law in emission as well as in immission on surfaces corresponds to spatial isotropy. In room acoustical computer simulation, the mix of diffuse and specular reflections in reality is often simulated by a 'diffusivity 'or 'scattering' coefficient usually interpolating between both cases drawing random numbers [2]. The scattering coefficient $\boldsymbol{\sigma}$ is defined as the proportion: non-geometrically energy'/total reflected energy'. A diffusivity coefficient may also include edge diffraction (often forgotten).

For the tolerant version of the definition of the DSF ('convergence only in late reverberation') it is sufficient that only an average absorption degree needs to be 'low', typically it is proposed that for the validity of the Sabine formula a mean absorption degree $\alpha_m < 0.3$ is sufficient and at least a small piece of surface is a bit unregularly i.e. scattering and hence 'mixing' [3]. Only if (fictively) the surfaces were also interchanging positions (evenly distributed), i.e. totally mixing (D2), then from C+D+D2 follows A+B +B2.

Relationship between spatial and temporal diffusivity (exponential decay)

Only if the room is 'totally mixing' i.e. interchanging energy at every place and time into every direction, then there is no chance that different (exponential) energy decays arise and just one single exponential energy decay with on reverberation time is left. A constant ***B*** is the condition that the notion 'equivalent absorption area' (used to derive the Sabine formula) i.e. a surface-weighting, makes sense.

3. Average quantities in a Diffuse sound field

The core physical quantity is the equivalent absorption area

$$A = \sum \alpha_i S_i \tag{2}$$

or the 'mean (surfaces averaged) absorption degree'

$$\alpha_m \equiv \alpha = \frac{\sum \alpha_i S_i}{S} = A/S \tag{3}$$

(α_i = absorption degrees, S_i= single of N surfaces, S= total surface, V=volume).
The other, the geometric average quantity, is the mean free path length with its famous formula

$$\Lambda = 4V/S \tag{4}$$

This formula is true even for non-convex rooms, if a diffuse sound field really were given – which is, however, hardly the case then. The correct mfp-formulae can be derived strictly obeying conditions A…D2:

Method a) is utilizing A) isotropy in ***Ω*** and **B)** homogeneity in V and averaging over the inverse mfp, i.e. reflection frequencies ('time average') (the dash – stands for averaging) [4].

$$\Lambda^{-1} = \overline{l^{-1}}^{V,\Omega} \tag{5}$$

In a DSF, 'sound particles' (sp) lose their identity: 'time = ensemble average' [1]. So, another

Method b) utilizes B2) i.e. constant irradiation of S, and **D)** (everywhere Lambert law) and direct averaging over the mfp ('ensemble average': one considers the 'fates' of different sps:

$$\Lambda = \overline{\mathrm{l\,p}'}^{\Omega,S} \tag{6}$$

Different from method a, method b is related to the surface related conditions for the DSF. Both methods result in $\Lambda = 4V/S$. Other results in literature are not strictly based on a DSF.

4. Re-derivation of reverberation formulae

Both reverberation formulae assume a diffuse sound field (especially condition B2 i.e. a constant irradiation of the surface leading to a mean absorption coefficient, E in Table 1.)

4.1. The Eyring formula

Typical is here to consider a ´representative sound particle` (sp) (Table 1, condition F).
This sp, after always a free path length Λ, 'sees' a surface with the absorption degree α_m.
The consequence is a stepwise exponential energy decay:

$$E(N) = E_0(1-\alpha_m)^N \quad \text{(E}_0\text{ = start energy, N=reflection number)} \tag{7}$$

By introducing a mean absorption exponent $\quad \alpha_m' = -\ln(1-\alpha_m)$ (8)

Eqn. 14 reads $E(N) = E_0 e^{-N\alpha_m'}$. For N reflections with a mfp Λ, the time $t = N\,\Lambda/c$ is needed. Tacitly it is assumed that ***N*** is a real number as after a switched off steady sound source the decays overlap and 'smooth' the resulting function $E(t) = E_0 e^{-\alpha_m' ct/\Lambda}$. Using the standard formulation of an exponential decay $E(t) = E_0 e^{-\frac{t}{\tau}}$ the time constant of the sound energy decay is

$$\tau_{ey} = \frac{\Lambda}{c\,\alpha_{m'}} \quad (9)$$

In such a **typical RT formula**, the time constant is always the proportion of the mean free path length and an average absorption exponent. The RT for a 60dB decay is then generally

$$T = 6\ln(10)\,\tau \quad (10)$$

Using the normalized value of the sound velocity c=340m/s at 14°C and also the value for the mfp Λ (Eqn. 7) yields the Eyring reverberation time

$$T_{ey} = \frac{6\ln(10)}{c}\frac{4V}{S\,\alpha_{m'}} \approx 0.163\,\frac{V}{S\,\alpha_{m'}} \quad (11)$$

So, additionally (to the DSF) it is assumed: D2): only with a total mixing the sp lose their identity; F): a representative sp may be assumed, and: 'the mfp are constant' (condition H).

4.2. The Sabine formula

The Sabine formula is not just an approximation of the Eyring formula. It has its own, amazingly different derivation. Neither the model of a sp nor the concept of a mfp is needed. Instead, the decay of the total sound energy E(t) is considered aiming at a differential equation. Especially the homogeneity of the energy distribution is assumed, even more, that there is simply only one value of ***E*** at the time ***t*** (condition G). This will turn out to be the **crucial misunderstanding of the Sabine approach**. With the energy density $\boldsymbol{U = E/V}$, $\boldsymbol{I = c\,U}$, the irradiation strength is $\boldsymbol{B = I/4 = cU/4 = cE/(4V)}$. ($I = \int j d\Omega$ = omnidirectional scalar intensity; '1/4' is due to the directional averaging of the projection factor $cos(\vartheta)$ (0 on backside) over the full solid angle 4π.)

Then the incident energy per time is

$$\frac{dE_i}{dt} = cE\frac{S}{(4V)} \quad (12),$$

and the absorbed energy

$$\frac{dE}{dt} = -E(t)\,c\frac{\alpha_m S}{4V} = -E(t)\,c\frac{\alpha_m}{\Lambda} \quad (13).$$

The solution is an exponential decay with the time constant

$$\tau_{sab} = \frac{\Lambda}{c\,\alpha_m} \quad (14)$$

Inserting again $\Lambda = 4V/S$ yields the famous Sabine RT

$$T_{sab} = 6 \cdot \ln(10)\,\tau_{sab} \approx 0.163\,\frac{V}{A} \quad (15).$$

As the energy is proportional to the number of sps, analogously to Eqn.12 the sp impact rate is

$$dN/dt = N/t = cN\,S/(4V) \quad (16)$$

which would be constant without absorption. After the time for travelling just a mfp, per definition all sp once have hit the room surface. Hence, inserting $t = \Lambda/c$ into Eqn. 16 yields by the way a prove for the formula for the mean free path length $\Lambda = 4V/S$.

5. Why is the Sabine different from the Eyring formula?

For small α_m is (by expansion)

$$\alpha_m' = -\ln(1-\alpha_m) \approx \alpha_m(1+\frac{\alpha_m}{2}) \quad (17).$$

So comparing both formulae (Eqns. 9 and 14) shows that the difference is just in the order of

$$\frac{\boldsymbol{T_{ey}}}{\boldsymbol{T_{sab}}} \approx (\boldsymbol{1-\alpha_m/2}) \quad (18).$$

The reasons for the difference are different tacit additional assumptions:
Eyring assumes a constant mean free path length and a stepwise energy decay, so as if all sound particles lose a part of their energy at the same time [6]. Sabine assumes only one energy value. This is absurd as this were only possible if the information about absorption of a sp at one part of the surface and thus a decrease of the total energy (density) in the room were spread infinitely fast to everywhere within the room, such that other sps would have suddenly less energy and therefore lose less energy at the next reflection- as if 'the sound particles know from each other'. This leads to an effectively smaller energy loss and hence a longer reverberation time than with just one 'representative' sp as with the Eyring theory.

6. Transitions between the two formulae

6.1. From Eyring to Sabine

Starting with the Eyring model, obviously one has to consider the time interval between two reflections $\Delta t = \Lambda/c$ more in detail. The first thinking model is to subdivide it. With an equally distributed time shift of many sound particles, the energy loss may be linearly interpolated, so, after a time $\Delta t/n$, assuming that perfect 'information and energy interchange' after each '1/n reflection' the energy loss factor is $(1-\alpha/n)$ After a whole reflection (n such steps) the energy would be multiplied by $(1-\alpha/n)^n$. For $n \to \infty$, the loss factor between two reflections becomes

$$f_{sab} = \lim_{n\to\infty}\left(1-\frac{\alpha}{n}\right)^n = e^{-\alpha} \approx 1-\alpha+\frac{1}{2}\alpha^2 \ldots \approx 1-\alpha\cdot(1-\frac{\alpha}{2}) \qquad (19).$$

$e^{-\alpha}$ is the Sabine energy loss factor for 1 reflection (insert $\tau = \frac{\Lambda}{c\,\alpha}$ and $t = \frac{\Lambda}{c}$ into $e^{-t/\tau}$). The corresponding Eyring value is $f_{Ey} = 1-\alpha = e^{-\alpha'}$ (20)

Comparing Eqn. 19 with 20, the 'effective' absorption degrees differ in the first approximation by the factor $(1-\alpha/2)$. So, the difference between the Eyring and the Sabine formula ($T_{ey}/\,T_{sab}$, equ.18) can be explained by the transition from the stepwise to a continuous absorption.

Here, it just shall be mentioned that Kuttruff found a formula to 'repair' Eyring's formula allowing varying the free path lengths. His approach [1] is to consider the reverberation as a sum of an infinite number of decays with different RTs. The effective absorption coefficient is then

$$\boldsymbol{\alpha''} = \boldsymbol{\alpha'}\left(\mathbf{1}-\boldsymbol{\gamma^2\alpha'}/\mathbf{2}\right) \text{ with } \boldsymbol{\gamma^2} = \frac{\bar{l^2}-\bar{l}^2}{\bar{l}^2} \qquad (21)$$

the relative variance of the free path lengths where $\bar{\mathbf{l}}$ is the mfp (called $\boldsymbol{\Lambda}$ before), $\bar{\mathbf{l}}^2$ its square and $\bar{\mathbf{l^2}}$ the average over the squares. This is smaller than the Eyring exponent $\boldsymbol{\alpha'}$. So, with varying free path lengths the RT is longer than without. One can see: The variation of reflection moments furthers the 'mixing effect' as it is, different from the Eyring theory, tacitly assumed within the derivation of the Sabine formula. For 'totally' varying free path lengths $\boldsymbol{\gamma^2} = \mathbf{1}$ and with the expansion $\boldsymbol{\alpha'} = -\ln(\mathbf{1}-\boldsymbol{\alpha}) \approx \boldsymbol{\alpha}+\boldsymbol{\alpha^2}/\mathbf{2}$ it turns out that $\boldsymbol{\alpha''} \approx \boldsymbol{\alpha}$. So, allowing totally varying free path lengths, the RT value of the Eyring formula converges against the Sabine value.

6.2. From Sabine to Eyring

An idea to describe the opposite transition is to assume that for the energy loss at the surface the total energy (considered with the Sabine model) in the middle of the room is relevant. Thus the former differential eqn.13 has to be altered to

$$\frac{dE}{dt} = -\frac{c\,\alpha}{\Lambda} E(t - \Delta t/2) \quad (22)$$

where $\Delta t = \frac{\Lambda}{c} = \alpha\tau_{sab}$ is the half of the time interval between two reflections. Assuming, as the first approximation, an exponential decay according the Sabine RT,

$$E\left(t - \frac{\Delta t}{2}\right) = E(t)e^{\Delta t/(2\tau sab)} = E(t)\, e^{\alpha/2} \approx E(t)\,(1 + \alpha/2) \quad (23).$$

Inserted into the differential eqn. 22 yields a differential eqn. with a modified absorption factor

$$\frac{dE}{dt} \approx -\frac{c\,\alpha\left(1+\frac{\alpha}{2}\right)}{\Lambda} E(t) = -E(t)/\tau_{sabshift} \quad (24).$$

The new time constant is $\tau_{sabshift} = \frac{\Lambda}{c\,\alpha(1+\alpha/2)} \approx \tau_{sab}(1 - \alpha/2) \approx \tau_{Ey}$ (25)

Thus, again the Eyring reverberation time is reached - explained by the forgotten time shift.

7. Analytical approaches for partially diffuse sound fields

In the following, some concepts shall be discussed which do not any longer assume homogeneous and/or isotropic sound fields, yet still diffuse reflections. As mentioned, even overall diffuse reflections do not guarantee a diffuse sound field, the irradiation strengths on the surfaces $\boldsymbol{B_i}$ may not be constant. A base to describe this is Kuttruff's integral equation for the irradiation strength $\boldsymbol{B(r,t)}$ [1]. The equation (already found by Clausius [5] for heat transfer) describes the radiation balance in a closed room with diffusely reflecting surfaces. This integral equation can only be solved numerically by the time dependent 'radiosity' method. A compromise is an iteration with the assumption of an approximately exponential decay where the reverberation time is delivered as an Eigenvalue [7]. A special application is the non-diffuse sound field of reverberation rooms causing wrong measurements of α.

7.1. Kuttruff's formula regarding the spreading of the absorption degrees

Aiming at an analytical approximation formula for a single exponential decay, Kuttruff [1] found a formula taking into account also the variance of the absorption degrees. All (still unknown) irradiation strengths $\boldsymbol{B_i}$ (of small discretized plane surfaces) are assumed to decay exponentially and all distances are replaced by the same mean free path length Λ. The approach starts with the idea of an 'effective', i.e. irradiation weighted new absorption exponent:

$$\alpha'' = -\ln\left(\frac{\sum_{i=1}^{K}\rho_i B_i S_i}{\sum_{i=1}^{K} B_i S_i}\right) \quad (\rho_n = \text{reflection degrees}) \quad (26).$$

Assuming, as a first guess, that the $\boldsymbol{B_i}$ are simply proportional to the other surfaces times their reflection coefficients, the result is:

$$\alpha'' = \alpha' + \ln\left(1 + \frac{\sum_{n=1}^{N}(\alpha_m - \alpha_n)(1-\alpha_n)S_n^2}{(1-\alpha_m)^2 \cdot S^2 - \sum_{n=1}^{N}(1-\alpha_n)^2 S_n^2}\right) \quad (27)$$

where the α_n are the absorption degrees of the surfaces, α_m their average, and α' is the Eyring absorption exponent. Analogously to Eyring is $\tau_{Kutt} = \Lambda/(c\,\alpha'')$ and $T_{Kutt} = 6\ln(10)\,\tau_{Kutt}$. For the frequent cases of one dominating absorbing surface (usually the floor), the term to the right in Eqn. 27 is positive, and the Kuttruff formula yields lower (typically 10-20% lower) RTs than according Eyring. So: $T_{Sab} > T_{Ey} > T_{Kutt}$.

7.2. Fitzroy's subdivision into separate reverb. processes in x-, y- and z-direction

Another early approach to take the special effects of a non-uniform absorption distribution into account, is that of Fitzroy [8]. Often, especially in planar rectangular rooms with few scattering and hence mixing, more or less separate reverberation processes establish in the three main axes' directions. Assigning for each direction (x,y,z) a specific mean absorption coefficient α_1, α_2 , α_3 and 'typical' mean free path lengths $\Lambda_1, \Lambda_2, \Lambda_3$, one could derive, in the same way as for the Eyring RT (section 4.1), specific reverberation times T_1, T_2 and T_3 respectively. Fitzroy's empirical compromise is just to take the arithmetic surface weighted average of all three:

$$T_{Fr} = \frac{S_1}{S} \cdot T_1 + \frac{S_2}{S} \cdot T_2 + \frac{S_3}{S} \cdot T_3 \qquad (28)$$

where the S_1, S_2 and S_3 are the surfaces of the room perpendicular to the x,y,z-direction. The weak point is: what are the values of the surfaces S_1, S_2 and S_3 in cases of non-rectangular rooms? Also, the choice of such three orthogonal directions may be quite arbitrary.

7.3. Arau's improved reverberation formula

The same unanswered questions apply to the model of Arau [9] who further developed Fitzroy's model. The basic approach of his formula is to account for different classes of reflections: $j = 1,2,3$ for x,y,z; N_j out of $N = \sum N_j$. In a DSF, i.e. with constant surface irradiation, the probabilities to hit a surface S_i are $p_i = S_i/S$, such that the reflection numbers seem to be $N_i = N \cdot p_i$. But, as Kuttruff showed, the p_i are just probabilities, the reflection numbers are not exactly N_i but the probability for e.g. N_1 reflections obeys a binomial distribution. Also Arau went this way of first introducing a binominal distribution going then over to a logarithmic normal distribution. First, a surface weighted average of the absorption exponent is applied to surfaces within the same class (e.g. opposite parallel walls):

$$\overline{\alpha_J} = \sum_i \mathbf{S_{Ji}/S_J \cdot \alpha_{Ji}} \qquad (29)$$

For the simultaneous reverberation processes, a stepwise decay as with the Eyring theory but with different steps according different classes of reflections, he derives the formula:

$$\overline{\alpha'_{Arau}} = \prod_{J=1}^{3} \left(\overline{\alpha'_J}\right)^{p_J} \qquad (30)$$

- an area weighted geometric mean of the weighted absorption exponents $\overline{\boldsymbol{\alpha}'_J} = -\boldsymbol{ln}\left(\mathbf{1} - \overline{\boldsymbol{\alpha}_J}\right)$ in the x,y,z- directions. Component wise insertion into the usual reverberation time formulae (like Eqns. 9,10) yields for the reverberation time finally an area weighted geometric mean:

$$T_{Arau} = T_x{}^{S_x/S} \cdot T_y{}^{S_y/S} \cdot T_z{}^{S_z/S} \qquad (31)$$

The computed results, obtained separately for early and late RT, agreed astonishingly well – and much better than Sabine and Eyring – with measurements – at least in a rectangular and highly diffusing hall. But: There is an important deficit at Arau's and Fitzroy's method: both seem to assume somehow geometric reflexions – but the decisive 'mixing' between the different reverberation classes caused by more or less scattering walls is not taken quantitatively into account.

So far, all RT formula assume totally diffuse reflections (some other, in rare cases, assume unrealistic totally geometric reflections as e.g. to estimate flutter-echoes).

7.4. The Nilsson/Gerretsen analytical model for rectangular rooms with partially diffusely reflecting surfaces

An innovative analytical approach is that by Nilsson [10]. His basic idea is to subdivide the reverberation process into a 'grazing' part (i.e. almost parallel to the ceiling) and a 'non-grazing' part. The coupling of these processes by scattering is derived from counting room modes and the Statistical Energy Analysis. Gerretsen [11] simplified Nilssons approach, translating it into the 'energetic language'. Instead of loss factors, he introduced the familiar notions of equivalent absorption or scattering areas. He generalizes the method to any absorption and scattering distribution: three 2-dimensional 'grazing' fields almost perpendicular to the x-, y.- and z-axes and an overall almost diffuse field for which 4 partial RTs and a RT for the total non-linear decay are derived.
The computational results applying these formulae are 'globally in accordance' with measured effects [11] for even and uneven absorption distribution (all walls reflecting, or just the ceiling or also the floor highly absorbing). For the uneven absorption distribution, the most interesting case here, the Sabine value is, as expected, much too low, the Fitzroy value much too high. The RT values from the Nilsson/Gerretsen model agree very well and best with the results from ray tracing with scattering. The Nilsson method has also been verified quite well by Prodi [12].

8. Semi-analytical models regarding scattering coefficients

As a compromise between simple analytic formulae and costly ray tracing methods, **an ‘Anisotropic Reverberation** Model’ [13] has been developed as a semi-analytical model that takes absorption and scattering coefficients into account as well as the orientation of the surfaces, however, not their positions. It assumes still a homogeneous but anisotropic sound field. The idea is to consider flowing sound energies in a group (typically some thousands) of angular ranges (like pyramidal beams but without defined starting vertices) and to define (as with the theory of coupled rooms) coefficients describing transitions between them over the relevant surfaces depending on their absorption and scattering coefficients and their orientation. This leads to a linear system of ordinary differential equations. This system can be solved either by iteration or with Eigenvalues (as partial RTs) and Eigenvectors describing energy distributions. Finally, specific early and late reverberation times and directivities are obtained.

Some semi-analytical procedures to compute RTs as a function of the scattering coefficients are presented in [2], however only in 2D:
1) For a semi-circular room with scattering ceiling and absorbing floor; here focusing effects may cause RTs much shorter than according Sabine, decreasing with decreasing scattering coefficients. An analytical formula is derived.
2) Reverberation in a 2D-rectangular room with absorbing side walls and reflecting front walls is handled by establishing an iteration scheme utilizing scattering dependent transition coefficients between the geometric and the rest diffuse sound field. Here – as tyoical for flutter echoes – RTs increase drastically (about inversely proportional) with decreasing scattering coefficients.
3) On the occasion of the Elbphilharmonie concert hall under construction in Hamburg – a vineyard hall -a drastic influence of scattering and inclination of the ceiling on the RT could be proved by sound particle simulation [14].

9. Conclusions

The "diffuse sound field" is a very idealistic assumption for zero absorption and total scattering and mixing - an utopia. So actually, both the Sabine and the Eyring formula are wrong. They must not be applied in many cases of non-diffuse reflections i.e. in many realistic cases. The reasons for the difference between the Sabine and the Eyring formula are tacit additional assumptions. For small absorption degrees, RTs lie between the Sabine value and the Eyring value, depending on the mfp distribution, described by the Kuttruff formula. The main condition in praxi is: reflections need to be sufficiently diffuse and absorption degrees not too unevenly distributed. A weak point is: scattering coefficients are often unknown. Most other reverberation formulae assume perfectly diffuse reflections yet yielding rather small corrections. They cannot explain effects of partly geometrical reflections which may be dominating in cases of focusing on reflecting or absorbing surfaces causing e.g. flutter echoes in shoe-box-rooms or focusing effects in domes. There are some analytical or semi-analytical approaches, however restricted to special geometries [2]. The only analytical model to compute RTs explicitly respecting scattering coefficients is that of Nilsson/Gerretsen - however restricted to rectangular rooms, published in the ISO 12354, part 6 - unfortunately only in the non- obligatory appendix D [15]. Reverberation times in non-diffuse sound fields depend on the room shape, the distribution of the absorption and especially the scattering coefficients. To compute such sound fields, numerical methods seem unavoidable.

References

[1] Kuttruff, H.: Room Acoustics, Elsevier Science Publishers Ltd., Barking, England

[2] Stephenson, U.M.: Are there simple reverberation time formulae also for partially diffusely reflecting surfaces ?; in: proc. of INTERNOISE, New York, 2012

[3] Joyce, W.B.; Sabine's reverberation time and ergodic auditoriums; J.Acoust.Soc.Am.58,3, (1975)

[4] Kosten,C.W.; The mean free path in room acoustics; Acustica 10 [1960], 245

[5] Clausius, R.; Die kinetische Theorie der Gase, 2. ed., (Vieweg, Braunschweig 1889), 51

[6] Cremer, L., Müller, H.; Principles and Applications of Room Acoustics, Vol. 1; translated by T.J. Schultz; Applied Science Publishers, London, New York 1978

[7] Kuttruff, H.; A Simple Iteration Scheme for the Computation of Decay Constants in Enclosures with Diffusely Reflecting Boundaries; J. Acoust.Soc. Am. 98 (1), 1995, 288

[8] Fitzroy, D; A reverberation formula which seems to be more accurate with a non-uniform distribution of absorption; J. Acoust. Soc. Amer. 31 [1959], p. 893

[9] Arau-Puchades, H.; An improved reverberation formula; Acustica 65 [1988], 163-180

[10] Nilsson, E., Decay process in rooms with non-diffuse sound fields, . Part I: Ceiling treatment with absorbing material; Part II: Effect of irregularities. Building Acoustics 11 (2004), 39-60, 133-144

[11] Gerretsen, E.; Estimation methods for sound levels and reverberation time in a room with irregular shape or absorption distribution; Acta Acustica United with Acustica 92 (2006), 797-806

[12] Prodi, N; et al. On the extended theory of sound fields in auditoria; in: proc. of AIA-DAGA; International conference on Acoustics; Merano, Italy, 2013

[13] Drechsler, S; The Anisotropic Reverberation Model (ARM) - a simplified method to calculate the sound energy decay and derived properties; in: proc. of INTERNOISE, New York, 2012

[14] Stephenson, U.M; On the influence of the ceiling and audience profile on the reverberation time and other room acoustical parameters; in: proc. of Intern. Conf. on Auditorium Acoustics 2008 Oslo; Inst. of Acoustics, University of Salford, UK, 2008

[15] EN 12354-6, Building acoustics – Estimation of acoustic performance of buildings from the performance of elements – Part 6: Sound absorption in enclosed spaces. CEN, Brussels, 2003