

## New ideas for a link between objective measurements and architecture in an ancient Italian Opera House

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It is usual in common people to think that ancient architects were deeply found of acoustic science, so ancient Opera Houses and Theatres in general were quite all acoustically perfect; but, when researchers try to find in literature what kind of acoustical study really has been carried on before Sabine, there is a deep delusion and only some generic assumptions are found about wood, shape, boxes structure and nothing more: for instance, for the rebuilding of La Fenice in Venice only some generic idea in the original manuscripts from a so famous architect like Meduna was found. On the other side, from the last century researchers in architectural acoustics are debating about the significance of certain objective parameters that can be measured utilising computerised techniques, special microphones and complex mathematical treatment of data, whose results need anyway an experimental confirmation from listeners on music.

### 1 Introduction

The refurbishing of a not too old Italian Opera House in Spoleto has given us the possibility for a deep study both on the theatre, making a lot of measurements, and on the drawings, making accurate geometrical measurements: also some old book written by ancient architects less known was picked up and studied having the target to understand why some acoustical defect was somewhere so easily listened in that theatre. Correlating some basic mathematical rule of musical harmony and the geometrical configuration of the hall, taking into account the results from acoustical measurements of parameters like reverberation time, ITDG, EDT and IACC, some interesting result has been found: this paper will relate on these subjects, while we are looking for some verification in other theatres nearby.

### 2 Basic ideas

#### 2.1 Previous theories about the best shape

In 1782 Pierre Patte wrote a book [1] dealing with acoustical considerations in drawing theatres and, starting from the idea that the ellipse must be the best shape for the plan design of the stalls, related with the best sound distribution, he derived his canonical rules to give right proportions to the hall; having in mind that boxes were a wooden made element added to the original flat hall to achieve new places in the theatre, where it was possible to place servants and people in general while nobility took place in the stalls, Patte

referred his ellipse to the back of the boxes, the masonry made structure (Figure 1).

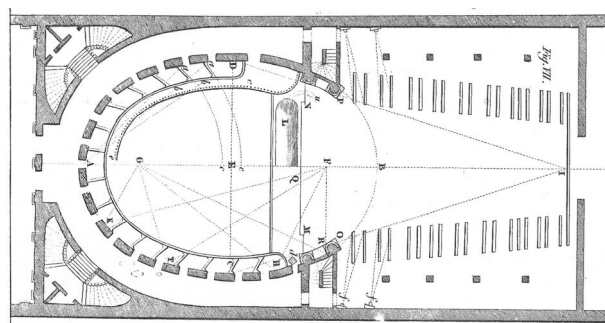


Figure 1: Geometrical acoustics applied to the sound rays computation in an ellipse stalls plan in accordance with the principles of Pierre Patte (1784).

Maybe, the origin of this kind of measurement was that boxes were built in wood within an existing hall with dimensions not originally foreseen. This was a mistake, as the acoustical field matches the front of boxes, not the walls of the hall, but anyway we can get some warning about the proportion between length, measured from the acoustical source, and the back of the hall at floor level, and width, measured along the normal axis of the ellipse, always at floor level in the hall: so, while in Patte's idea this ratio must be about 1.2, in Figure 1 at the hall floor it is about 1. Patte gives also some warning about the height of the hall, measured from the stage floor, that must not exceed the width of the hall, measured from the back of the opposite boxes, that is to say about 0.8 of the real floor width: he says also that the section must not be too concave to avoid redundancies of sound. Carini [2] says that the height must be within 1 and 2/3 of the width of the hall, again measured in the largest place of

the floor. So, we have found in literature that assuming 1 the height, width must be between 1.5 and 1.25 but preferably 1.5 [2], and assuming 1 the width, the length must be also 1 [1].

## 2.2 Recent theories about the best shape

In contemporary literature, like Sepmeyer [3], Louden [3], Volkman [3], Boner [4], Beranek [5], we can found other ratios, that are presented in Table 1.

Table 1: Suggested dimensional ratios for the best modal distribution in rectangular halls and corresponding number of overlapping modes

		Height	Width	Length	N° of O. modes
Sepmeyer	A	1.00	1,14	1,39	9
Sepmeyer	B	1.00	1,28	1,54	10
Sepmeyer	C	1.00	1,6	2,33	9
Louden	D	1.00	1,4	1,9	11
Louden	E	1.00	1,3	1,9	8
Louden	F	1.00	1,5	2,5	12
Volkman	G	1.00	1,5	2,5	12
Boner	H	1.00	1,26	1,59	11
Beranek	I	1.00	1,6	2,3	9
Beranek	L	1.00	2	3	10
Cesare 1	M	1.00	1,49	2,98	6
Cesare 2	N	1.00	1,49	2,22	7

If one utilises these ratios to evaluate the natural modes of a rectangular room, limiting the frequency between 0 Hz and 50 Hz, it is possible to see that each of them presents some modes very close one to the other, sometimes even overlapping, and this situation means the possibility to have beating waves: it is well known that beating waves are responsible for unpleasant listening of music.

This means that we can test the degree of preference of a hall on the basis of the proportions of the hall itself, as they can be responsible of beating waves.

Our idea is that these ratios must be connected also with the musical theory, so we have tested the mayor scales in their 12 tonalities comparing them with 9 imaginary halls deriving their proportions in plant from the main musical ratios (5°, 4°, etc.) and searching, among them, the ratio giving the lowest number of dissonances. For instance, one of these tests is represented in Table 2. Not reporting here the details of this research, it was found that the best ratios were 1.33 and 1.49 [6], as shown in the last two rows of Table 1.

This result is not astonishing, as they are the well known ratios of forth and fifth: but if we test these ratios looking for the natural modes in a room as before, one can see that we obtain the lesser number of

overlapping modes, as presented in Figure 2 related to the arrangement "M".

Table 2: an example of analysis of 12 tonalities of mayor scales with an imaginary hall of dimensions in plant whose dimensions are in ratio 1.49

Tab. D - 4 tonalità su 12 risultano in dissonanza -								Rapporto = 1,49		Tonalità in dissonanza
1°	2°	3°	4°	5°	6°	7°	8°	Do	Sol	
Do	Re	Mi	Fa	Sol	La	Si	Do	1°	5°	
Do#	Re#	Fa	Fa#	Sol#	La#	Do	Do#	7°	5°-	X
Re	Mi	Fa#	Sol	La	Si	Do#	Re	7°-	4°	
Re#	Fa	Sol	Sol#	La#	Do	Re	Re#	6°	3°	
Mi	Fa#	Sol#	La	Si	Do#	Re#	Mi	5°+	3°-	X
Fa	Sol	La	La#	Do	Re	Mi	Fa	5°	2°	
Fa#	Sol#	La#	Si	Do#	Re#	Fa	Fa#	5°-	1°+	X
Sol	La	Si	Do	Re	Mi	Fa#	Sol	4°	1°	
Sol#	La#	Do	Do#	Re#	Fa	Sol	Sol#	3°	7°	
La	Si	Do#	Re	Mi	Fa#	Sol#	La	3°-	7°-	
La#	Do	Re	Re#	Fa	Sol	La	La#	2°	6°	
Si	Do#	Re#	Mi	Fa#	Sol#	La#	Si	1°+	5°+	X

The conclusion of this particular research is that Patte and Carini were closest to that values and this may be one of the reasons for ancient theatres are acoustically good.

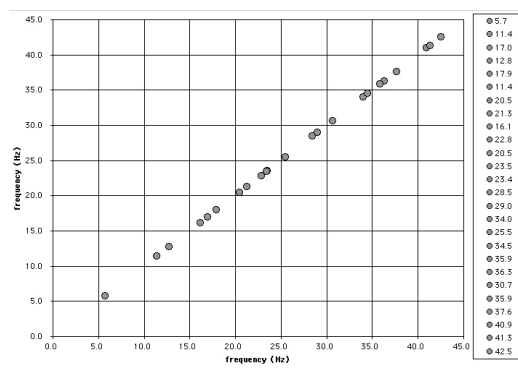


Figure 2: Modal distribution related to our proposal "M" of Table 1.

## 2.3 The Teatro Nuovo in Spoleto

As reported in [7], one of the authors was charged to take an acoustical image of the theatre before the beginning of its refurbishing works: this was a big opportunity for taking so many measurements as it was possible, changing either the position of sources or the position of receivers: during the measurements' campaigns it was clear that there was an acoustical defect that interested only a portion of the stalls, that is to say a flutter echo picked up directly listening, but

not clearly pointed out by the usual objective parameters like EDT, T30, C50, C80, G, STI, ITDG, IACC, LF.

Only from the examination of EDT in the frequency domain it seems possible to identify some architectural elements eventually responsible of some wrong dimension in the theatre: this particular aspect is now under deep examination.

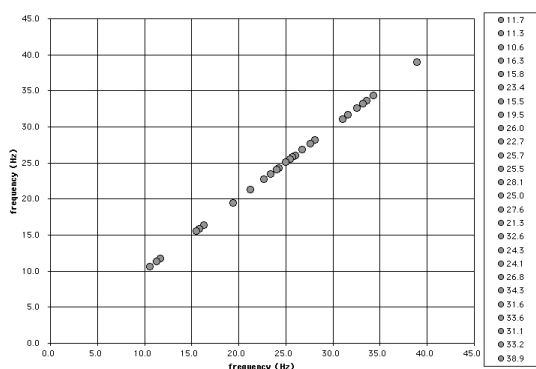


Figure 3: Modal distribution realised in a rectangular hall of the main dimensions of Teatro Nuovo,

The possibility of having such a defect is instead clearly evidenced by the architectural measures of simple distances as above: height, width, and length are here 16m, 15m e 14.5m that is to say 1.00:0.93:0.90, so that there can be present many overlapping modes as shown in Figure 3.

An Italian theatre usually differs from a rectangular shape and the relative surfaces following x, y and z directions are rounded, so many natural modes of Figure 3 are not effective in the real hall, but not all are destroyed. So, through the reading of the IR in the frequency domain [6] it is possible to pick up some particular mode.

### 3 IR in the Frequency Domain

#### 3.1 Basic specifications about the IR frequency analysis

In this section the results relative to the analysis of the IR (Impulse Response) in the frequency domain will be presented for the source positioned in the front of the pit and the receiver positioned in the stalls.

Figure 4 presents some of the results related to the first 70 ms from the arrival of the direct signal at the receiver; these 4 images are spaced in time by 20 ms each other and cover frequencies between 40 Hz and 1 kHz; each sample is the mean of the signal received within 10 ms, that is to say 5 ms before and 5 ms after

the target. The black lines and the grey lines in the spectrograms in Figure 4 refer to left and right channel respectively and the black and grey arrows point out the peaks respectively; for more specifications about this technique of analysis, please refer to [6].

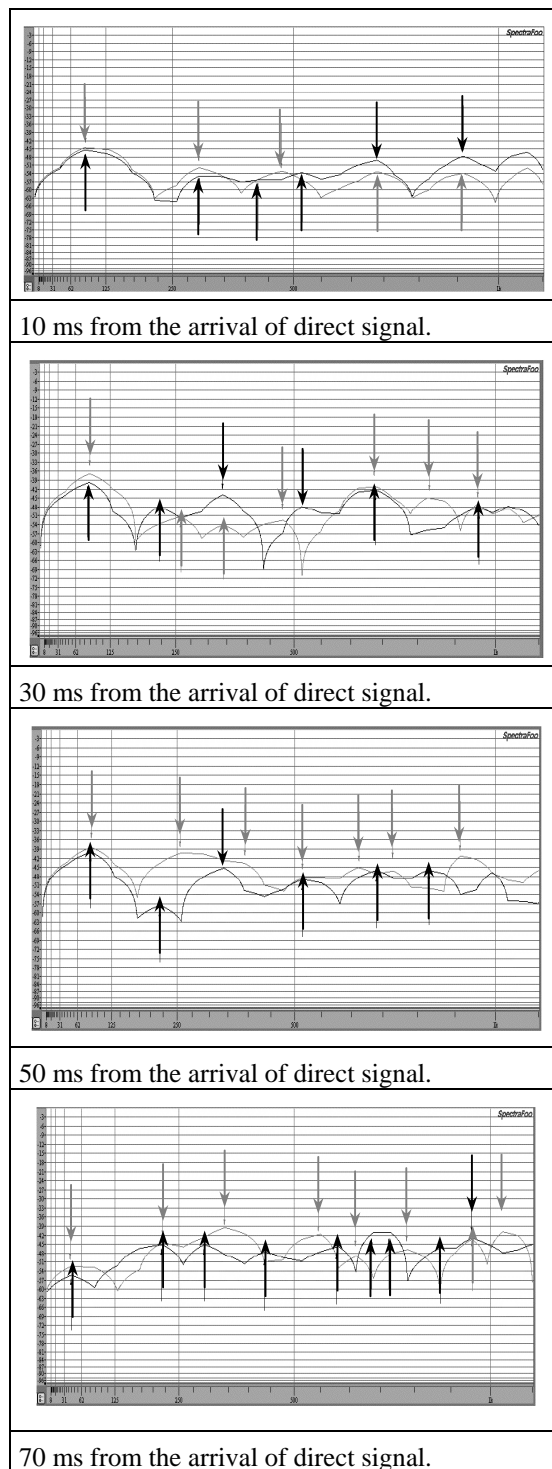


Figure 4: Spectrograms related to the four time selected intervals

Table 3: Presentation of the results coming from the analysis of IR in the frequency domain, source in the front of the pit, receiver in the stalls.

Position	1 (10 ms)		2 (30 ms)		3 (50 ms)		4 (70 ms)	
	L	R	L	R	L	R	L	R
Peak frequencies or Modes (Hz)	<b>86</b>	<b>86</b>	<b>86</b>	<b>86</b>	<b>86</b>	<b>86</b>	<b>43</b>	<b>43</b>
	<i>301</i>	<i>301</i>	<i>215</i>	<i>258</i>	<i>215</i>	<i>258</i>	<i>215</i>	<i>215</i>
	<b>430</b>	<b>473</b>	<b>344</b>	<b>344</b>	<b>344</b>	<i>387</i>	<i>301</i>	<b>344</b>
	<b>516</b>	<b>688</b>	<b>516</b>	<i>473</i>	<b>516</b>	<b>516</b>	<b>430</b>	<i>559</i>
	<b>688</b>	<i>903</i>	<b>688</b>	<b>688</b>	<b>688</b>	<b>645</b>	<b>602</b>	<i>645</i>
	<i>903</i>		<b>946</b>	<i>817</i>	<i>817</i>	<i>731</i>	<b>688</b>	<b>774</b>
				<b>946</b>		<b>946</b>	<i>731</i>	<b>946</b>
							<b>860</b>	
							<b>946</b>	

From Figure 4 it's possible to derive the data collected in Table 3: note that some frequencies are repeated constantly and anyway that they are harmonics of 43 Hz and 86 Hz. In Table 3 the multiple frequencies of 43 Hz only are in *italic* font, while the multiple frequencies of 86 Hz are in **bold** font; of course the 86 Hz is a multiple of 43 Hz, but as we'll see later in this section, it's better to keep them distinguished.

As we can see in Table 3, the first 3 time intervals present the fundamental frequency of 86 Hz, while the last one starts with the fundamental frequency of 43 Hz. The target of this analysis is to compare between the fundamental frequency and its harmonics with the dimensions of the environment where the measurement campaign was carried out (Teatro Nuovo), through an accurate analysis of axial modes.

As mentioned above, the source of this IR has been positioned in the pit whose walls are not parallel, but ceiling and floor run parallel with height of 2 m and area of 140 m<sup>2</sup>, so it represents the largest area with a rectangular shape in this theatre; then for  $n_x = 1$  and  $c = 344 \text{ ms}^{-1}$ , the result will be:

$$f_1 = \frac{c}{2} \left( \frac{n_x}{l_x} \right) = 86 \text{ Hz} \quad (1)$$

For the next modes, let's say  $n_x = 2, 3, \dots, 6$ , we'll have the harmonic frequencies  $f_2 = 172 \text{ Hz}$ ,  $f_3 = 258 \text{ Hz}$ , ...,  $f_6 = 516 \text{ Hz}$ . So, the frequency of the first vertical mode gives the same result as coming from the IR; is it a coincidence? If not, going ahead with this kind of analysis, the maximum dimension of the proscenium arch is around 14 m and so for  $n_x = 7$  the

corresponding frequency will be  $f_7 = 86 \text{ Hz}$ . It is worthwhile to mention that our software acoustically exciting the hall is based on a logarithmically sine-swept signal starting at 40 Hz, so the up mentioned  $f_7$  is for us the forth recordable harmonic of this mode.

As already known, it's possible to say that any theatre is a system based on coupled spaces: here we have two of them (the pit section covered by the stage and that covered by the proscenium arch) that are linked one to the other by the vertical dimension (1 to 7), having both a correspondence with a particular frequency, well evidenced in the decaying curve.

Another dimension that we can consider here is the bending radius of the ceiling, corresponding to 24 m; for  $n_x = 6$  in (1), the result will be  $f_6 = 43 \text{ Hz}$ . As just said before, for our system this is the first recordable harmonic of this mode. Moreover, the maximum distance between the ceiling and the stalls is 16 m and for  $n_x = 4$  the corresponding frequency is exactly  $f_4 = 43 \text{ Hz}$ , again the first one available in our records.

The reason why in the first 3 time intervals of Table 3 the first frequency corresponds to 86 Hz, while in the forth we begin with 43 Hz is clear if one takes into consideration the time factor; in fact the stationary waves corresponding to the ceiling and recorded by the receiver placed in the stalls are delayed in time by the major distance, in comparison with the minor one between pit and stalls; for this reason, the receiver in the stalls records the first stationary wave coming from pit (86 Hz) and after 60-70 ms, he records the stationary wave coming from the ceiling (43 Hz).

## 4 Consequences

Having the opportunity for changing some elements while restoring the Opera House, we have tried to get some teaching from the above described analysis.

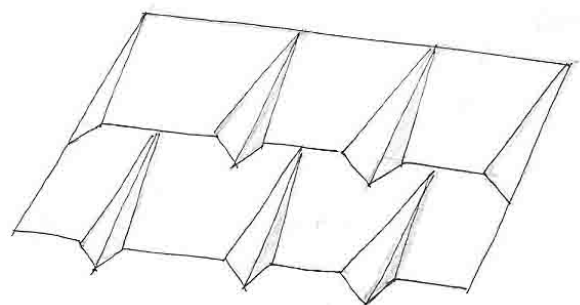


Figure 5: the suggested shape for the pit' ceiling

The floor of the pit will be lowered of about 0.45 m, so its ceiling having the height of the proscenium arch (now 14.45 m) will be no more a multiple of pit height. The ceiling of the pit will be no more flat, but enlivened with inclined surfaces, with a mean height of

about 2.15 – 2.20 m) as presented in the sketch of Figure 5.

As we think that the flutter echo is due to the delay deriving from the strong bending radius of the ceiling, a

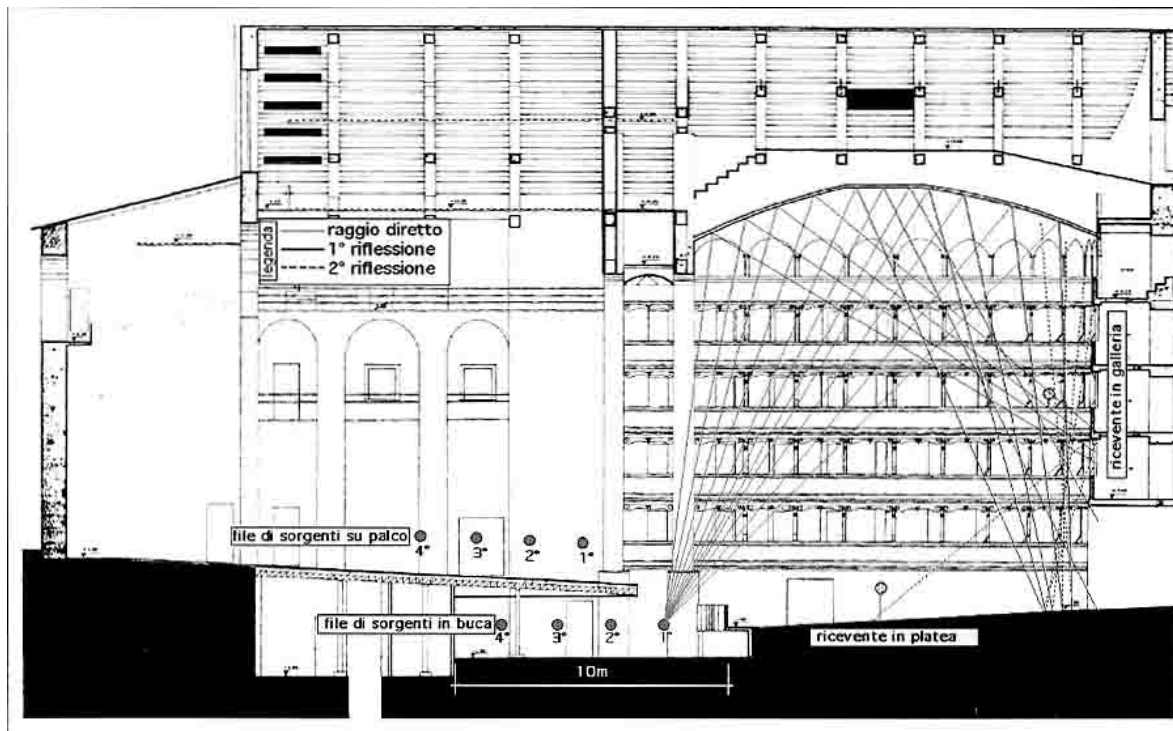


Figure 6: Geometrical construction of sound rays starting from the front pit and received in the stalls with the original ceiling

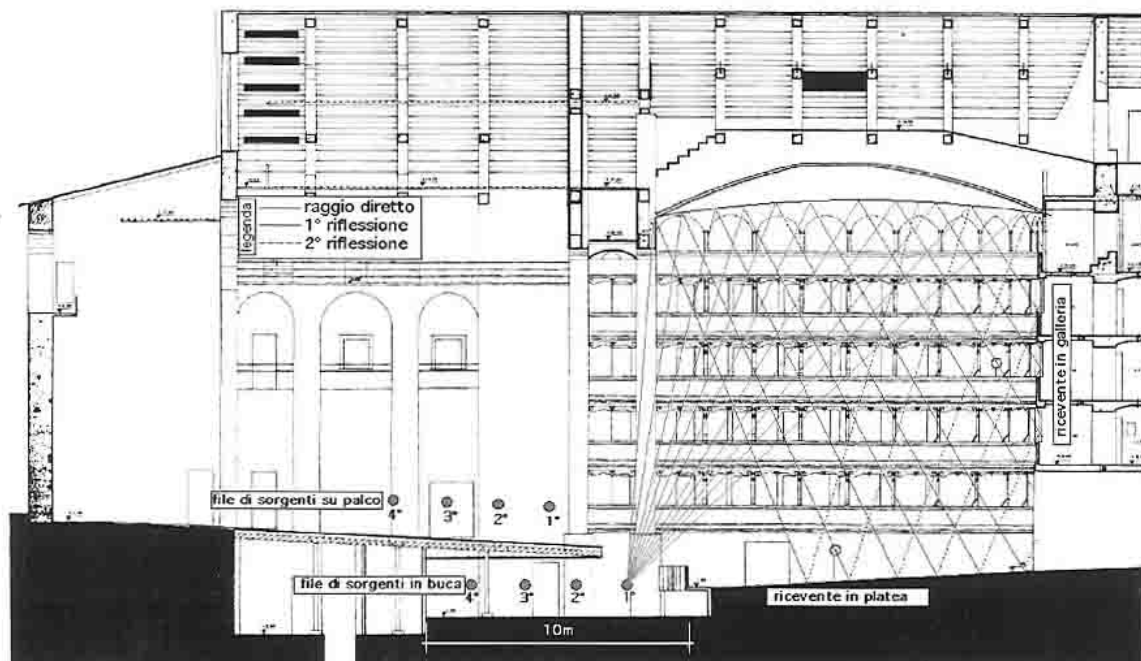


Figure 7: Geometrical construction of sound rays starting from the front pit and received in the stalls with the ceiling modified

geometrical change has been checked, and it is presented in the following Figures 6 and 7.

In Figure 6 it is evident the possibility for some ray to reach the receiver in the stalls with a remarkable time delay, while in Figure 7 the rays coming from the ceiling reach directly the stalls.

As it is not possible to destroy the original painted ceiling, we propose a particular kind of annular structure, in a transparent material and lightly convex, to be put in place and removed with winches: in this way, the result of changing the height of the hall will also be achieved.

## 5 Summary

With the measurement campaign in Teatro Nuovo in Spoleto [8], it has been possible to analyse this environment through the acoustical objective parameters [7] and also through a new method based on comparison among ancient acoustical theories (1600-1800), architecture, acoustics and modal theory [6]. Having in mind the theory about the asymmetrical distribution of absorption [9], it has been demonstrated that also for large environments (Italian opera houses) a part of the modal theory (axial modes) appears to be confirmed even in elliptical shapes by the analysis of the IR in the frequency domain, to the best of our knowledge carried out for the first time in an Italian theatre [6]. These results need further verifications through more measurements in other theatres, whose findings will be made available as soon as possible.

It is well known that the analysis of the geometry of the hall can help to find a solution to acoustical problems [10] even before it will be possible to carry out measurements in a model or in a real hall: in this case we have analysed an already existing hall, and we have found the origin of an acoustical defect starting from measurements and looking for coincidence of geometrical dimensions and related modes. At the end of this analysis it has been possible to put forward some suggestion about possible geometrical modifications consistent with the conservation of the original building as a cultural heritage.

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